3-1 Symmetry and Coordinate Graphs

Page 133 Graphing Calculator Exploration

1. $f(-x) = f(x)$ **2.** $f(-x) = -f(x)$

3. even; odd

4. $f(x) = x^8 - 3x^4 + 2x^2 + 2$

$f(-x) = (-x)^8 - 3(-x)^4 + 2(-x)^2 + 2$
$= x^8 - 3x^4 + 2x^2 + 2$
$= f(x)$

$f(x) = x^7 + 4x^5 - x^3$

$f(-x) = (-x)^7 + 4(-x)^5 - (-x)^3$
$= -x^7 - 4x^5 + x^3$
$= -(x^7 + 4x^5 - x^3)$
$= -f(x)$

5. First find a few points of the graph in either the first or fourth quadrants. For an even function, a few other points of the graph are found by using the same y-values as those points, but with opposite x-coordinates. For an odd function, a few other points are found by using the opposite of both the x- and y-coordinates as those original points.

6. By setting the INDPNT menu option in TBLSET to ASK instead of AUTO**,** you can then go to TABLE and input x-values and determine their corresponding y-values on the graph. By inputting several sets of opposite pairs, you can observe whether $f(-x) = f(x)$, $f(-x) = -f(x)$, or neither of these relationships is apparent.

Pages 133–134 Check for Understanding

1. The graph of $y = -x^2 + 12$ is an even function. The graph of $xy = 6$ is an odd function. The graphs of $x = y^2 - 4$ and $17x^2 + 16xy + 17y^2 = 225$ are neither.

2. The graph of an odd function is symmetric with respect to the origin. Therefore, rotating the graph 180° will have no effect on its appearance. See student's work for example.

3a. Sample answer: $y = 0$, $x = 0$, $y = x$, $y = -x$

3b. infinitely many

3c. point symmetry about the origin

4. Substitute (a, b) into the equation. Substitute $(-b, -a)$ into the equation. Check to see whether both substitutions result in equivalent equations.

5. Alicia

Graphically: If a graph has origin symmetry, then any portion of the graph in Quadrant I has an image in Quadrant III. If the graph is then symmetric with respect to the y-axis, the portion in Quadrants I and II have reflections in Quadrants II and IV, respectively. Therefore, any piece in Quadrant I has a reflection in Quadrant IV and the same is true for Quadrants II and III. Therefore, the graph is symmetric with respect to the x-axis.

Algebraically: Substituting $(-x, -y)$ into the equation followed by substituting $(-x, y)$ is the same as substituting $(x, -y)$.

6. $f(x) = x^6 + 9x$

$f(-x) = (-x)^6 + 9(-x)$ $\quad -f(x) = -(x^6 + 9x)$

$f(-x) = x^6 - 9x$ $\quad -f(x) = -x^6 - 9x$

no

7. $f(x) = \frac{1}{5x} - x^{19}$

$f(-x) = \frac{1}{5(-x)} - (-x)^{19}$ $\quad -f(x) = -\left(\frac{1}{5x} - x^{19}\right)$

$f(-x) = -\frac{1}{5x} + x^{19}$ $\quad -f(x) = -\frac{1}{5x} + x^{19}$

yes

8. $6x^2 = y - 1$ $\rightarrow$ $6a^2 = b - 1$

x-axis: $6a^2 = (-b) - 1$; $6a^2 = -b - 1$ no

y-axis: $6(-a)^2 = b - 1$; $6a^2 = b - 1$ yes

$y = x$: $6(b)^2 = a - 1$; $6b^2 = a - 1$ no

$y = -x$: $6(-b)^2 = (-a) - 1$; $6b^2 = -a - 1$ no

y-axis

9. $x^3 + y^3 = 4$ $\rightarrow$ $a^3 + b^3 = 4$

x-axis: $a^3 + (-b)^3 = 4$; $a^3 - b^3 = 4$ no

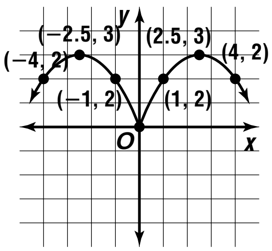
y-axis: $(-a)^3 + b^3 = 4$; $-a^3 + b^3 = 4$ no

$y = x$: $(b)^3 + (a)^3 = 4$; $a^3 + b^3 = 4$ yes

$y = -x$: $(-b)^3 + (-a)^3 = 4$; $-a^3 - b^3 = 4$ no

$y = x$

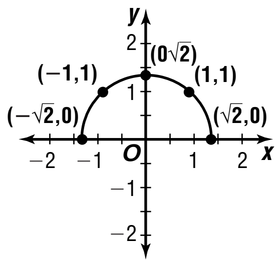
10.



11. $y = \sqrt{2 - x^2}$ $\rightarrow$ $b = \sqrt{2 - a^2}$

x-axis: $-b = \sqrt{2 - a^2}$ no

y-axis: $b = \sqrt{2 - (-a)^2}$; $b = \sqrt{2 - a^2}$ yes



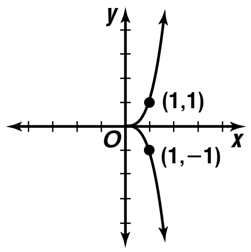
12. $|y| = x^3$ $\rightarrow$ $|b| = a^3$

x-axis $|-b| = a^3$

$|b| = a^3$ yes

y-axis $|b| = (-a)^3$

$|b| = -a^3$ no



13. x-intercept:

$$\frac{x^2}{25} - \frac{y^2}{9} = 1$$
$$\frac{x^2}{25} - \frac{0^2}{9} = 1$$
$$\frac{x^2}{25} = 1$$
$$x^2 = 25$$
$$x = \pm 5 \quad (5, 0)$$

other points:

when $x = 6$

$$\frac{36}{25} - \frac{y^2}{9} = 1$$
$$-\frac{y^2}{9} = -\frac{11}{25}$$
$$y^2 = \frac{99}{25}$$
$$y = \pm\frac{3\sqrt{11}}{5}$$

when $x = -6$

$$\frac{36}{25} - \frac{y^2}{9} = 1$$
$$-\frac{y^2}{9} = -\frac{11}{25}$$
$$y^2 = \frac{99}{25}$$
$$y = \pm\frac{3\sqrt{11}}{5}$$

$\left(6, -\frac{3\sqrt{11}}{5}\right), \left(-6, -\frac{3\sqrt{11}}{5}\right), \left(-6, \frac{3\sqrt{11}}{5}\right)$

Pages 134–136 Exercises

14. $f(x) = 3x$

$f(-x) = 3(-x)$ $\quad -f(x) = -(3x)$

$f(-x) = -3x$ $\quad -f(x) = -3x$

yes

15. $f(x) = x^3 - 1$

$f(-x) = (-x)^3 - 1$ $\quad -f(x) = -(x^3 - 1)$

$f(-x) = -x^3 - 1$ $\quad -f(x) = -x^3 + 1$

no

16. $f(x) = 5x^2 + 6x + 9$

$f(-x) = 5(-x)^2 + 6(-x) + 9$

$f(-x) = 5x^2 - 6x + 9$

$-f(x) = -(5x^2 + 6x + 9)$

$-f(x) = -5x^2 - 6x - 9$

no

17. $f(x) = \frac{1}{4x^7}$

$f(-x) = \frac{1}{4(-x)^7}$ $\quad -f(x) = -\left(\frac{1}{4x^7}\right)$

$f(-x) = -\frac{1}{4x^7}$ $\quad -f(x) = -\frac{1}{4x^7}$

yes

18. $f(x) = -7x^5 + 8x$

$f(-x) = -7(-x)^5 + 8(-x)$

$f(-x) = 7x^5 - 8x$

$-f(x) = -(-7x^5 + 8x)$

$-f(x) = 7x^5 - 8x$

yes

19. $f(x) = \frac{1}{x} - x^{100}$

$f(-x) = \frac{1}{(-x)} - (-x)^{100}$ $\quad -f(x) = -\left(\frac{1}{x} - x^{100}\right)$

$f(-x) = -\frac{1}{x} - x^{100}$ $\quad -f(x) = -\frac{1}{x} + x^{100}$

no

20. yes;

$g(x) = \frac{x^2 - 1}{x}$

$g(-x) = \frac{(-x)^2 - 1}{(-x)}$ *Replace x with −x.*

$g(-x) = -\frac{x^2 - 1}{x}$ $\quad (-x)^2 = x^2$

$-g(x) = -\left(\frac{x^2 - 1}{x}\right)$ *Determine the opposite of the function.*

$g(-x) = -g(x)$

21. $xy = 5$ $\rightarrow$ $ab = -5$

x-axis $a(-b) = -5$

$-ab = -5$

$ab = 5$ no

y-axis $(-a)b = -5$

$-ab = -5$

$ab = 5$ no

$y = x$ $(b)(a) = -5$

$ab = -5$ yes

$y = -x$ $(-b)(-a) = -5$

$ab = -5$ yes

$y = x$ and $y = -x$

22. $x + y^2 = 1$ $\rightarrow$ $a + b^2 = 1$

x-axis $a + (-b)^2 = 1$

$a + b^2 = 1$ yes

y-axis $(-a) + b^2 = 1$

$-a + b^2 = 1$ no

$y = x$ $(b) + (a)^2 = 1$

$b + a^2 = 1$ no

$y = -x$ $(-b) + (-a)^2 = 1$

$-b + a^2 = 1$ no

x-axis

23. $y = -8x$ $\rightarrow$ $b = -8a$

x-axis $(-b) = -8a$

$-b = -8a$ no

y-axis $b = -8(-a)$

$b = 8a$ no

$y = x$ $(a) = -8(b)$

$a = -8b$ no

$y = -x$ $(-a) = -8(-b)$

$-a = 8b$ no

none of these

24. $y = \frac{1}{x^2} \quad \rightarrow \quad b = \frac{1}{a^2}$

x-axis $\quad (-b) = \frac{1}{a^2}$

$-b = \frac{1}{a^2}$ no

y-axis $\quad b = \frac{1}{(-a)^2}$

$b = \frac{1}{a^2}$ yes

$y = x \quad (a) = \frac{1}{(b)^2}$

$a = \frac{1}{b^2}$ no

$y = -x \quad (-a) = \frac{1}{(-b)^2}$

$-a = \frac{1}{b^2}$ no; y-axis

25. $x^2 + y^2 = 4 \quad \rightarrow \quad a^2 + b^2 = 4$

x-axis $\quad a^2 + (-b)^2 = 4$

$a^2 + b^2 = 4$ yes

y-axis $\quad (-a)^2 + b^2 = 4$

$a^2 + b^2 = 4$ yes

$y = x \quad (b)^2 + (a)^2 = 4$

$a^2 + b^2 = 4$ yes

$y = -x \quad (-b)^2 + (-a)^2 = 4$

$a^2 + b^2 = 4$ yes

all

26. $y^2 = \frac{4x^2}{9} - 4 \quad \rightarrow \quad b^2 = \frac{4a^2}{9} - 4$

x-axis $\quad (-b)^2 = \frac{4a^2}{9} - 4$

$b^2 = \frac{4a^2}{9} - 4$ yes

y-axis $\quad b^2 = \frac{4(-a)^2}{9} - 4$

$b^2 = \frac{4a^2}{9} - 4$ yes

$y = x \quad (a)^2 = \frac{4(b)^2}{9} - 4$

$a^2 = \frac{4b^2}{9} - 4$ no

$y = -x \quad (-a)^2 = \frac{4(-b)^2}{9} - 4$

$a^2 = \frac{4b^2}{9} - 4$ no

x-axis and y-axis

27. $x^2 = \frac{1}{y^2} \quad \rightarrow \quad a^2 = \frac{1}{b^2}$

x-axis $\quad a^2 = \frac{1}{(-b)^2}$

$a^2 = \frac{1}{b^2}$ yes

y-axis $\quad (-a)^2 = \frac{1}{b^2}$

$a^2 = \frac{1}{b^2}$ yes

$y = x \quad (b)^2 = \frac{1}{(a)^2}$

$b^2 = \frac{1}{a^2}$

$a^2 = \frac{1}{b^2}$ yes

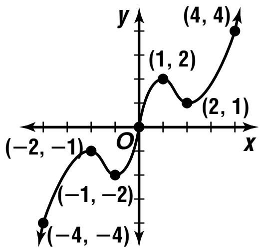
$y = -x \quad (-b)^2 = \frac{1}{(-a)^2}$

$b^2 = \frac{1}{a^2}$

$a^2 = \frac{1}{b^2}$ yes

x-axis, y-axis, $y = x$, $y = -x$

28.



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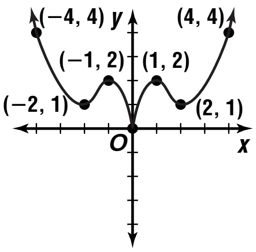
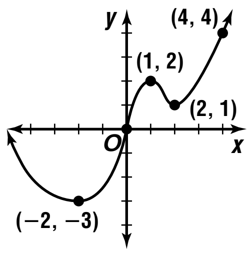


30. Sample answer:



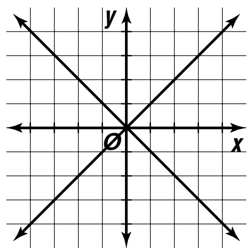
31. $y^2 = x^2 \quad \rightarrow \quad b^2 = a^2$

x-axis $\quad (-b)^2 = a^2$

$b^2 = a^2$

y-axis $\quad b^2 = (-a)^2$

$b^2 = a^2$ yes; both



32. $|x| = -3y \quad \rightarrow \quad |a| = -3b$

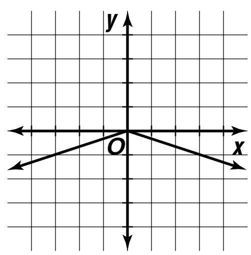
x-axis $\quad |a| = -3(-b)$

$|a| = 3b$ no

y-axis $\quad |(-a)| = -3b$

$|a| = -3b$ yes

y-axis



33. $y^2 + 3x = 0 \rightarrow b^2 + 3a = 0$

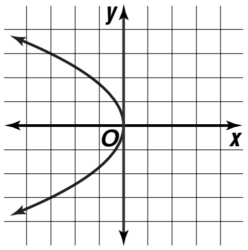
x-axis: $(-b)^2 + 3a = 0$

$b^2 + 3a = 0$ yes

y-axis: $b^2 + 3(-a) = 0$

$b^2 - 3a = 0$ no

x-axis



34. $|y| = 2x^2 \rightarrow |b| = 2a^2$

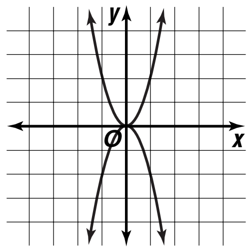
x-axis: $|(-b)| = 2a^2$

$|b| = 2a^2$ yes

y-axis: $|b| = 2(-a)^2$

$|b| = 2a^2$

yes; both



35. $x = \pm\sqrt{12 - 8y^2} \rightarrow a = \pm\sqrt{12 - 8b^2}$

x-axis: $a = \pm\sqrt{12 - 8(-b)^2}$

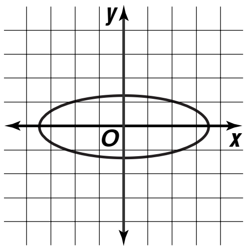
$a = \pm\sqrt{12 - 8b^2}$

yes

y-axis: $(-a) = \pm\sqrt{12 - 8b^2}$

$a = \pm\sqrt{12 - 8b^2}$

yes; both



36. $|y| = xy \rightarrow |b| = ab$

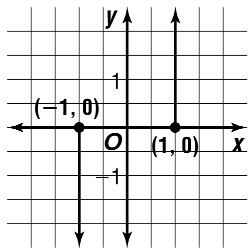
x-axis: $|(-b)| = a(-b)$

$|b| = -ab$ no

y-axis: $|b| = (-a)b$

$|b| = -ab$ no

neither



37. $|y| = x^3 - x \rightarrow |b| = a^3 - a$

x-axis: $|(-b)| = a^3 - a$

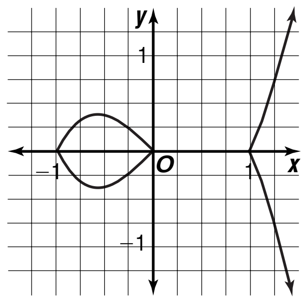
$|b| = a^3 - a$ yes

y-axis: $|b| = (-a)^3 - (-a)$

$|b| = -a^3 + a$ no

x-axis

The equation $|y| = x^3 - x$ is symmetric about the x-axis.



38a. $\frac{x^2}{8} + \frac{y^2}{10} = 1 \rightarrow \frac{a^2}{8} + \frac{b^2}{10} = 1$

x-axis: $\frac{a^2}{8} + \frac{(-b)^2}{10} = 1$

$\frac{a^2}{8} + \frac{b^2}{10} = 1$ yes

y-axis: $\frac{(-a)^2}{8} + \frac{b^2}{10} = 1$

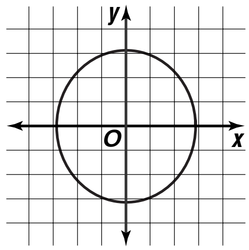
$\frac{a^2}{8} + \frac{b^2}{10} = 1$ yes

origin: $\frac{(-a)^2}{8} + \frac{(-b)^2}{10} = 1$

$\frac{a^2}{8} + \frac{b^2}{10} = 1$ yes

x- and y-axis symmetry

38b.



38c. $(2, -\sqrt{5}), (-2, \sqrt{5}), (-2, -\sqrt{5})$

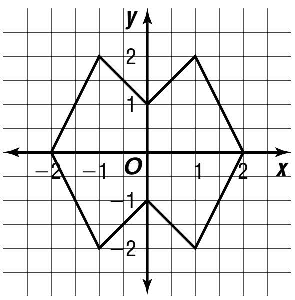
39. Sample answer: $y = 0$

40. Sample answer:

$y = x + 1$ $\quad y = -x + 1$ $\quad y = x - 1$

$y = -x - 1$ $\quad y = -2x + 4$ $\quad y = 2x + 4$

$y = -2x - 4$ $\quad y = 2x - 4$



41. $\frac{y^2}{12} - \frac{x^2}{16} = 1$

$\frac{(6)^2}{12} - \frac{x^2}{16} = 1$

$3 - \frac{x^2}{16} = 1$

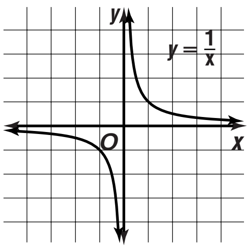
$-\frac{x^2}{16} = -2$

$x^2 = 32$

$x = \pm 4\sqrt{2}$

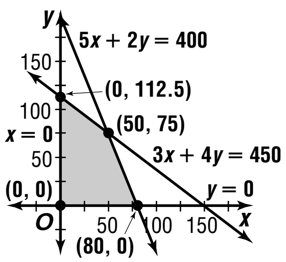
$(4\sqrt{2}, 6)$ or $(-4\sqrt{2}, 6)$

42. No; if an odd function has a y-intercept, then it must be the origin. If it were not, say it were (0, 1), then the graph would have to contain (−1, 0). This would cause the relation to fail the vertical line test and would therefore not be a function. But, not all odd functions have a y-intercept. Consider the graph of $y = \frac{1}{x}$.



43. Let x = number of bicycles.
Let y = number of tricycles.

$3x + 4y \leq 450$
$5x + 2y \leq 400$
$x \geq 0$
$y \geq 0$



$P(x, y) = 6x + 4y$
$P(0, 0) = 6(0) + 4(0)$ or 0
$P(0, 112.5) = 6(0) + 4(112.5)$ or 450
$P(50, 75) = 6(50) + 4(75)$ or 600
$P(80, 0) = 6(80) + 4(0)$ or 480

50 bicycles, 75 tricycles

44. $\begin{bmatrix} 4 & 3 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 8 & 5 \\ 9 & 6 \end{bmatrix} = \begin{bmatrix} 4(8) + 3(9) & 4(5) + 3(6) \\ 7(8) + 2(9) & 7(5) + 2(6) \end{bmatrix}$

$= \begin{bmatrix} 59 & 38 \\ 74 & 47 \end{bmatrix}$

45. $3(2x + y + z) = 3(0)$
$3x - 2y - 3z = -21$
$\rightarrow$
$6x + 3y + 3z = 0$
$\underline{3x - 2y - 3z = -21}$
$9x + y = -21$

$3x - 2y - 3z = -21$
$\underline{4x + 5y + 3z = -2}$
$7x + 3y = -23$

$-3(9x + y) = -3(-21)$
$7x + 3y = -23$
$\rightarrow$
$-27x - 3y = 63$
$\underline{7x + 3y = -23}$
$-20x = 40$
$x = -2$

$9x + y = -21$
$9(-2) + y = -21$
$y = -3$

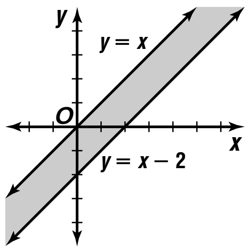
$2x + y + z = 0$
$2(-2) + (-3) + z = 0$
$z = 7$

$(-2, -3, 7)$

46. $4x - 2y = 7$
$-12x + 6y = -21$
$\rightarrow$
$y = 2x - \frac{7}{2}$
$y = 2x - \frac{7}{2}$

consistent and dependent

47.



48. $m = \frac{16 - 2}{-2 - 0}$

$= \frac{14}{-2}$ or -7

$y - 2 = -7(x - 0)$
$y = -7x + 2$

49. $[f \circ g](x) = f(g(x))$
$= f(x - 6)$
$= -2(x - 6) + 11$
$= -2x + 23$

$[g \circ f](x) = g(f(x))$
$= g(-2x + 11)$
$= (-2x + 11) - 6$
$= -2x + 5$

50. $75^3 \cdot 75^7 = 75^{3+7}$
$= 75^{10}$

The correct choice is B.

3-2 Families of Graphs

Page 142 Check for Understanding

1. $y = (x + 4)^3 - 7$

2. The graph of $y = (x + 3)^2$ is a translation of $y = x^2$ three units to the left. The graph of $y = x^2 + 3$ is a translation of $y = x^2$ three units up.

3. reflections and translations

4. When $c > 1$, the graph of $y = f(x)$ is compressed horizontally by a factor of c.
When $c = 1$, the graph of $y = f(x)$ is unchanged.
When $0 < c < 1$, the graph is expanded horizontally by a factor of $\frac{1}{c}$.

5a. $g(x) = \sqrt[3]{x} + 1$

5b. $h(x) = -\sqrt[3]{x - 1}$

5c. $k(x) = \sqrt[3]{x + 2} + 1$

6. The graph of $g(x)$ is the graph of $f(x)$ translated left 4 units.

7. The graph of $g(x)$ is the graph of $f(x)$ compressed horizontally by a factor of $\frac{1}{3}$, and then reflected over the x-axis.

8a. expanded horizontally by a factor of 5

8b. translated right 5 units and down 2 units

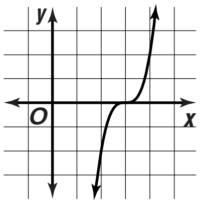
8c. expanded vertically by a factor of 3, translated up 6 units

9a. translated up 3 units, portion of graph below x-axis reflected over the x-axis

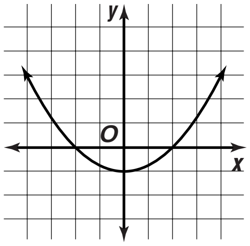
9b. reflected over the x-axis, compressed horizontally by a factor of $\frac{1}{2}$

9c. translated left 1 unit, compressed vertically by a factor of 0.75

10.

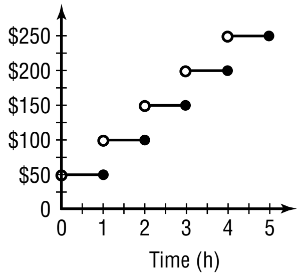


11.

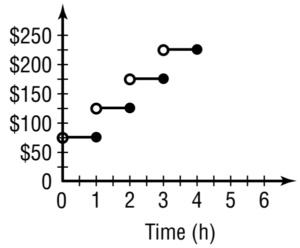


12a.

x	$f(x)$
$0 < x \le 1$	50
$1 < x \le 2$	100
$2 < x \le 3$	150
$3 < x \le 4$	200
$4 < x \le 5$	250



12b.



12c. $225

Pages 143–145 Exercises

13. The graph of $g(x)$ is a translation of the graph of $f(x)$ up 6 units.

14. The graph of g(x) is the graph of f(x) compressed vertically by a factor of $\frac{3}{4}$.

15. The graph of $g(x)$ is the graph of $f(x)$ compressed horizontally by a factor of $\frac{1}{5}$.

16. The graph of $g(x)$ is a translation of $f(x)$ right 5 units.

17. The graph of $g(x)$ is the graph of $f(x)$ expanded vertically by a factor of 3.

18. The graph of $g(x)$ is the graph of $f(x)$ reflected over the x-axis.

19. The graph of $g(x)$ is the graph of $f(x)$ reflected over the x-axis, expanded horizontally by a factor of 2.5, and translated up 3 units.

20a. reflected over the x-axis, compressed horizontally by a factor of $0.\overline{6}$

20b. translated right 3 units, expanded vertically by a factor of 4

20c. compressed vertically by a factor of $\frac{1}{2}$, translated down 5 units

21a. expanded horizontally by a factor of 5

21b. expanded vertically by a factor of 7, translated down 0.4 units

21c. reflected across the x-axis, translated left 1 unit, expanded vertically by a factor of 9

22a. translated left 2 units and down 5 units

22b. expanded horizontally by a factor of 1.25, reflected over the x-axis

22c. compressed horizontally by a factor of $\frac{3}{5}$, translated up 2 units

23a. translated left 2 units, compressed vertically by a factor of $\frac{1}{3}$

23b. reflected over the y-axis, translated down 7 units

23c. expanded vertically by a factor of 2, translated right 3 units and up 4 units

24a. expanded horizontally by a factor of 2

24b. compressed horizontally by a factor of $\frac{1}{6}$, translated 8 units up

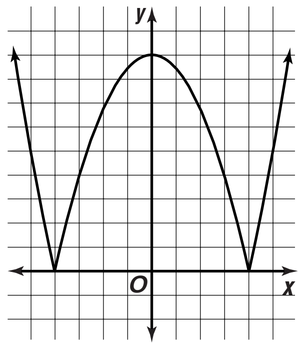
24c. The portion of parent graph on the left of the y-axis is replaced by a reflection of the portion on the right of the y-axis.

25a. compressed horizontally by a factor of $\frac{2}{5}$, translated down 3 units

25b. reflected over the y-axis, compressed vertically by a factor of 0.75

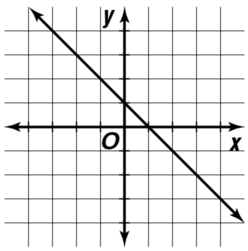
25c. The portion of the parent graph on the left of the y-axis is replaced by a reflection of the portion on the right of the y-axis. The new image is then translated 4 units right.

26. $y = x^2$



27. $y = \frac{0.25}{x - 4} + 3$

28.



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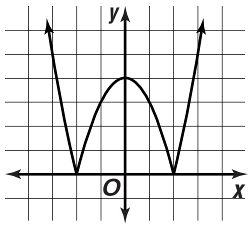
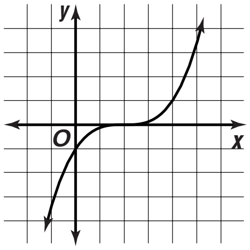


30.



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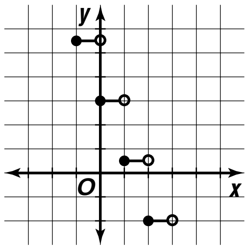

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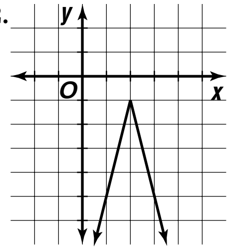

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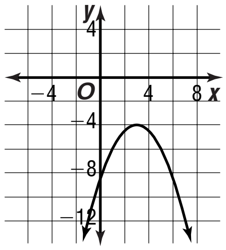

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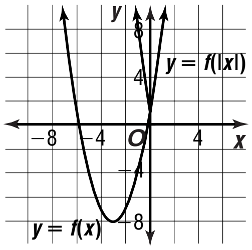
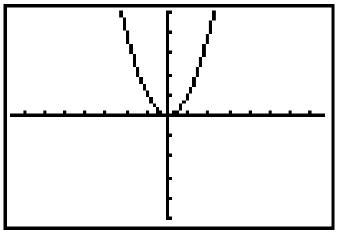
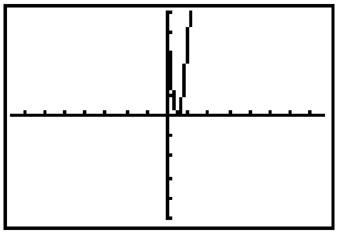


35a. 0



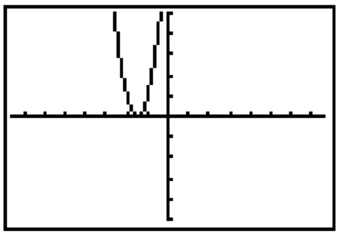
[−7.6, 7.6] scl:1 by [−5, 5] scl:1

35b. 0.5



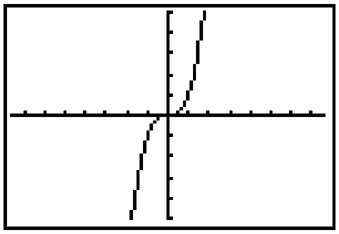
[−7.6, 7.6] scl:1 by [−5, 5] scl:1

35c. −1.5



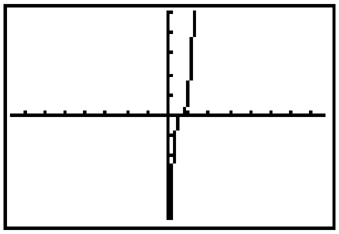
[−7.6, 7.6] scl:1 by [−5, 5] scl:1

36a. 0



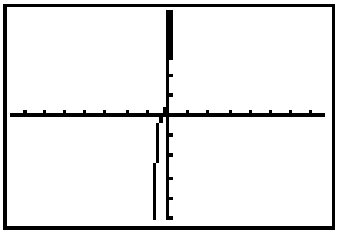
[−7.6, 7.6] scl:1 by [−5, 5] scl:1

36b. $0.6\overline{6}$



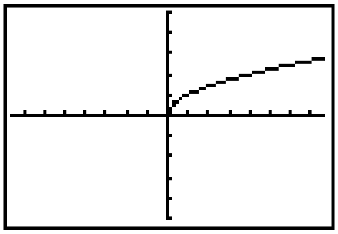
[−7.6, 7.6] scl:1 by [−5, 5] scl:1

36c. −0.25



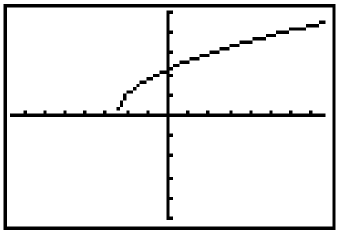
[−7.6, 7.6] scl:1 by [−5, 5] scl:1

37a. 0



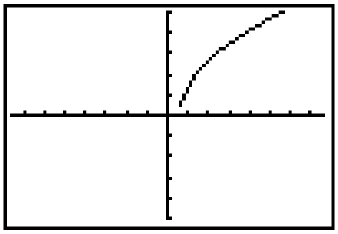
[−7.6, 7.6] scl:1 by [−5, 5] scl:1

37b. −2.5



[−7.6, 7.6] scl:1 by [−5.5] scl:1

37c. 0.6



[−7.6, 7.6] scl:1 by [−5, 5] scl:1

38a. The graph would continually move left 2 units and down 3 units.

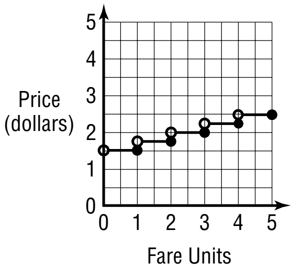
38b. The graph would continually be reflected over the x-axis and moved right 1 unit.

39. The x-intercept will be $-\frac{b}{a}$.

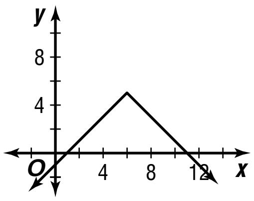
40a. $y = \begin{cases} 0.25[[x-1]] + 1.50 \text{ if } [[x]] = x \\ 0.25[[x]] + 1.50 \text{ if } [[x]] < x \end{cases}$

40b.

x	y
$0 < x \le 1$	1.50
$1 < x \le 2$	1.75
$2 < x \le 3$	2.00
$3 < x \le 4$	2.25
$4 < x \le 5$	2.50

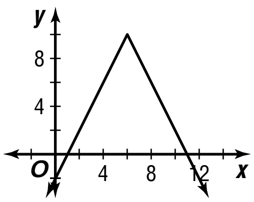


41a.

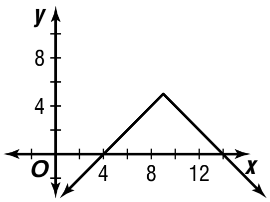


$A = \frac{1}{2}bh$
$= \frac{1}{2}(10)(5)$
$= 25 \text{ units}^2$

41b. The area of the triangle is $A = \frac{1}{2}(10)(10)$ or 50 units^2. Its area is twice as large as that of the original triangle. The area of the triangle formed by $y = c \cdot f(x)$ would be $25c$ units^2.

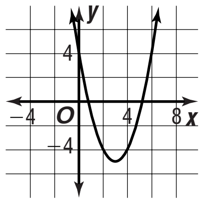


41c. The area of the triangle is $A = \frac{1}{2}(10)(5) =$ 25 units^2. Its area is the same as that of the original triangle. The area of the triangle formed by $y = f(x + c)$ would be 25 units^2.

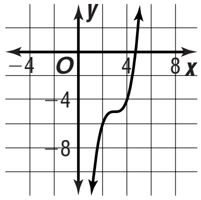


42a. (1) $y = x^2$ (2) $y = x^3$
(3) $y = -x^2$ (4) $y = -x^3$

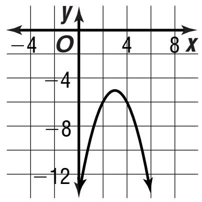
42b. (1)



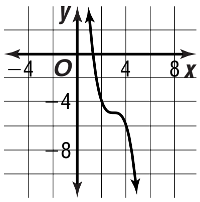
(2)



(3)



(4)



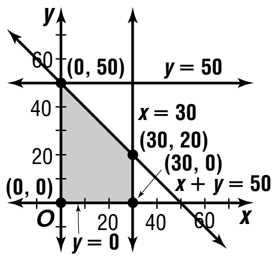
42c. (1) $y = (x-3)^2 - 5$ (2) $y = (x-3)^3 - 5$
(3) $y = -(x-3)^2 - 5$ (4) $y = -(x-3)^3 - 5$

43a. reflection over the x-axis, reflection over the y-axis, vertical translation, horizontal compression or expansion, and vertical expansion or compression

43b. horizontal translation

44. $f(x) = x^{17} - x^{15}$
$f(-x) = (-x)^{17} - (-x)^{15}$ $\quad -f(x) = -(x^{17} - x^{15})$
$f(-x) = -x^{17} + x^{15}$ $\quad -f(x) = -x^{17} + x^{15}$
yes; $f(-x) = -f(x)$

45. Let x = number of preschoolers.
Let y = number of school-age children.
$x + y \le 50$
$x \le 3(10)$
$y \le 5(10)$
$x \ge 0$
$y \ge 0$



$I(x, y) = 18x + 6y$
$I(0, 0) = 18(0) + 6(0)$ or 0
$I(0, 50) = 18(0) + 6(50)$ or 300
$I(30, 20) = 18(30) + 6(20)$ or 660
$I(30, 0) = 18(30) + 6(0)$ or 540
30 preschoolers and 20 school-age

46. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 5 & 1 & -2 \\ -4 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -3 & 1 \\ 5 & 1 & -2 \end{bmatrix}$
$A'(4, 5)$, $B'(-3, 1)$, $C'(1, -2)$

47. $x^2 = 25$ $\quad 9 = y$ $\quad 12 = 2z$
$x = \pm 5$ $\quad\quad 6 = z$

48. $-5(6x + 5y) = -5(-14)$ → $-30x - 25y = 70$
$6(5x + 2y) = 6(-3)$ $\quad \underline{30x + 12y = -18}$
$-13y = 52$
$y = -4$

$5x + 2y = -3$
$5x + 2(-4) = -3$
$x = 1$ $\quad (1, -4)$

49. The graph implies a negative linear relationship.

50. $3x - 4y = 0$ → $y = \frac{3}{4}x$
perpendicular slope: $-\frac{4}{3}$

51. $5d - 2p = 500 \quad \rightarrow \quad p = \frac{5}{2}d - 250$
-250

52. $[f \circ g](x) = f(g(x))$
$= f(x^2 - 6x + 9)$
$= \frac{2}{3}(x^2 - 6x + 9) - 2$
$= \frac{2}{3}x^2 - 4x + 4$

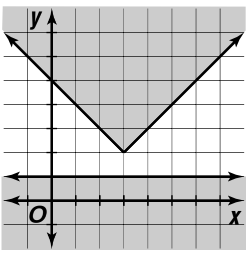
$[g \circ f](x) = g(f(x))$
$= g\left(\frac{2}{3}x - 2\right)$
$= \left(\frac{2}{3}x - 2\right)^2 - 6\left(\frac{2}{3}x - 2\right) + 9$
$= \frac{4}{9}x^2 - \frac{8}{3}x + 4 - 4x + 12 + 9$
$= \frac{4}{9}x^2 - \frac{20}{3}x + 25$

53. If $m = 1$; $d = 1 - \frac{50}{1}$ or -49.
If $m = 10$; $d = 10 - \frac{50}{10}$ or 5.
If $m = 50$; $d = 50 - \frac{50}{50}$ or 49.
If $m = 100$; $d = 100 - \frac{50}{100}$ or 99.5
If $m = 1000$; $d = 1000 - \frac{50}{1000}$ or 999.95.
The correct choice is A.

3-3 Graphs of Nonlinear Inequalities

Page 149 Check for Understanding

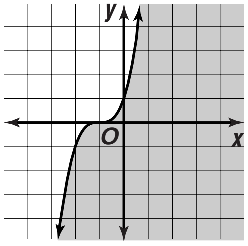
1. A knowledge of transformations can help determine the graph of the boundary of the shaded region, $y = 5 + \sqrt{x - 2}$.
2. When solving a one variable inequality algebraically, you must consider the case where the quantity inside the absolute value is non-negative and the case where the quantity inside the absolute value is negative.
3. Sample answer: Pick a point not on the boundary of the inequality, and test to see whether it is a solution to the inequality. If that point is a solution, shade all points in that region. If it is not a solution to the inequality, test a point on the other side of the boundary and shade accordingly.
4. This inequality has no solution since the two graphs do not intersect



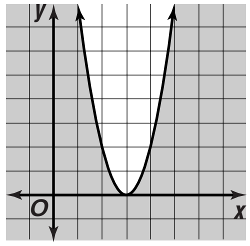
5. $y \geq -5x^4 + 7x^3 + 8$
$-3 \overset{?}{\geq} -5(-1)^4 + 7(-1)^3 + 8$
$-3 \geq -4$; yes

6. $y < |3x - 4| - 1$
$3 \overset{?}{<} |3(0) - 4| - 1$
$3 \not< 3$; no

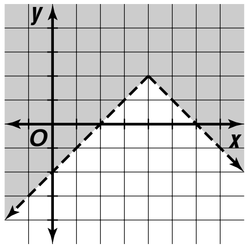
7.



8.



9.



10. Case 1
$|x + 6| > 4$
$-(x + 6) > 4$
$-x - 6 > 4$
$-x > 10$
$x < -10$
$\{x \mid x < -10 \text{ or } x > -2\}$

Case 2
$|x + 6| > 4$
$x + 6 > 4$
$x > -2$

11. Case 1
$|3x - 4| \leq x$
$-(3x - 4) \leq x$
$-3x + 4 \leq x$
$-4x \leq -4$
$x \geq 1$
$\{x \mid 1 \leq x \leq 2\}$

Case 2
$|3x - 4| \leq x$
$3x - 4 \leq x$
$2x \leq 4$
$x \leq 2$

12a. $|x - 12| \leq 0.005$

12b. Case 1
$|x - 12| \leq 0.005$
$-(x - 12) \leq 0.005$
$-x + 12 \leq 0.005$
$-x \leq -11.995$
$x \geq 11.995$
12.005 cm, 11.995 cm

Case 2
$|x - 12| \leq 0.005$
$x - 12 \leq 0.005$
$x \leq 12.005$

Pages 150–151 Exercises

13. $y < x^3 - 4x^2 + 2$
$0 \overset{?}{<} (1)^3 - 4(1)^2 + 2$
$0 \not< -1$; no

14. $y < |x - 2| + 7$
$8 \overset{?}{<} |3 - 2| + 7$
$8 \not< 8$; no

15. $y > -\sqrt{x + 11} + 1$
$-1 \overset{?}{>} -\sqrt{(-2) + 11} + 1$
$-1 > -2$; yes

16. $y < -0.2x^2 + 9x - 7$
$63 \overset{?}{<} -0.2(10)^2 + 9(10) - 7$
$63 \not< 63$; no

17. $y \leq \frac{x^2 - 6}{x}$
$-9 \overset{?}{\leq} \frac{(-6)^2 - 6}{-6}$
$-9 \leq -5$; yes

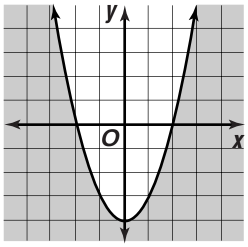
18. $y \geq 2|x|^3 - 7$
$0 \overset{?}{\geq} 2|0|^3 - 7$
$0 \geq -7$; yes

19. $y \le \sqrt{x} + 2$
$0 \overset{?}{\le} \sqrt{0} + 2$
$0 \le 2$; yes
$y \le \sqrt{x} + 2$
$1 \overset{?}{\le} \sqrt{1} + 2$
$1 \le 3$; yes
$y \le \sqrt{x} + 2$
$-1 \overset{?}{\le} \sqrt{1} + 2$
$-1 \le 3$; yes

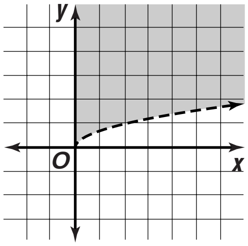
$y \le \sqrt{x} + 2$
$4 \overset{?}{\le} \sqrt{1} + 2$
$4 \not\le 3$; no
$y \le \sqrt{x} + 2$
$0 \overset{?}{\le} \sqrt{-1} + 2$
$0 \not\le \boldsymbol{i} + 2$; no

(0, 0) (1, 1) and (1, −1); if these points are in the shaded region and the other points are not, then the graph is correct.

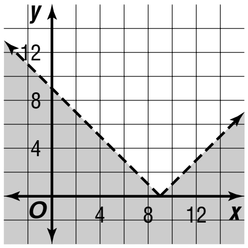
20.



21.



22.



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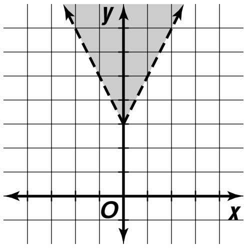
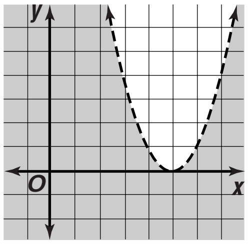
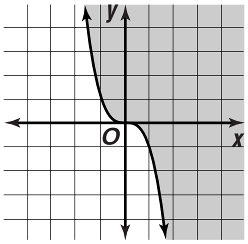


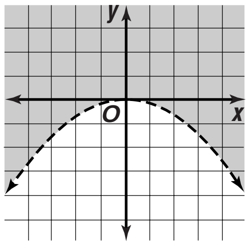
24.



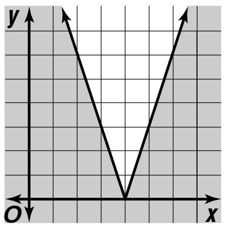
25.



26.



27.



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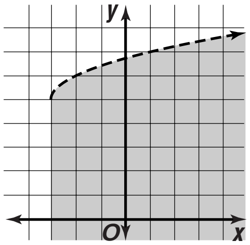

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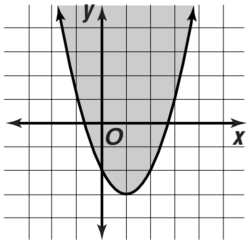

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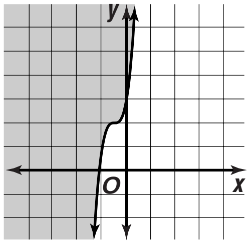
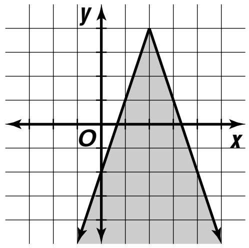
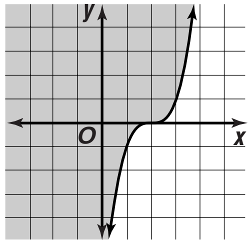


31.



32.



33. Case 1
$|x + 4| > 5$
$-(x + 4) > 5$
$-x - 4 > 5$
$-x > 9$
$x < -9$

Case 2
$|x + 4| > 5$
$x + 4 > 5$
$x > 1$

$\{x \mid x < -9 \text{ or } x > 1\}$

34. Case 1
$|3x + 12| \ge 42$
$-(3x + 12) \ge 42$
$-3x - 12 \ge 42$
$-3x \ge 54$
$x \le -18$

Case 2
$|3x + 12| \ge 42$
$3x + 12 \ge 42$
$3x \ge 30$
$x \ge 10$

$\{x \mid x \le -18 \text{ or } x \ge 10\}$

35. Case 1
$|7 - 2x| - 8 < 3$
$-(7 - 2x) - 8 < 3$
$-7 + 2x - 8 < 3$
$2x < 18$
$x < 9$

Case 2
$|7 - 2x| - 8 < 3$
$7 - 2x - 8 < 3$
$-2x < 4$
$x > -2$

$\{x \mid -2 < x < 9\}$

36. Case 1
$|5 - x| \le x$
$-(5 - x) \le x$
$-5 + x \le x$
$-5 \le 0$; true

Case 2
$|5 - x| \le x$
$5 - x \le x$
$-2x \le -5$
$x \ge 2.5$

$\{x \mid x > 2.5\}$

37. Case 1
$|5x - 8| < 0$
$-(5x - 8) < 0$
$-5x + 8 < 0$
$-5x < -8$
$x > \frac{8}{5}$

Case 2
$|5x - 8| < 0$
$5x - 8 < 0$
$5x < 8$
$x > \frac{8}{5}$

no solution

38. Case 1
$|2x + 9| - 2x \ge 0$
$-(2x + 9) - 2x \ge 0$
$-2x - 9 - 2x \ge 0$
$-4x \ge 9$
$x \le -\frac{9}{4}$

Case 2
$|2x + 9| - 2x \ge 0$
$2x + 9 - 2x \ge 0$
$9 \ge 0$; true

all real numbers

39. Case 1

$$-\tfrac{2}{3}|x+5| \geq -8$$
$$-\tfrac{2}{3}(-(x+5)) \geq -8$$
$$-\tfrac{2}{3}(-x-5) \geq -8$$
$$\tfrac{2}{3}x + \tfrac{10}{3} \geq -8$$
$$\tfrac{2}{3}x \geq -\tfrac{34}{3}$$
$$x \geq -17$$

Case 2

$$-\tfrac{2}{3}|x+5| \geq -8$$
$$-\tfrac{2}{3}(x+5) \geq -8$$
$$-\tfrac{2}{3}x - \tfrac{10}{3} \geq -8$$
$$-\tfrac{2}{3}x \geq -\tfrac{14}{3}$$
$$x \leq 7$$

$\{x \mid -17 \leq x \leq 7\}$

40. $|x - 37.5| \leq 1.2$

Case 1

$$|x - 37.5| \leq 1.2$$
$$-(x - 37.5) \leq 1.2$$
$$-x + 37.5 \leq 1.2$$
$$-x \leq -36.3$$
$$x \geq 36.3$$

Case 2

$$|x - 37.5| \leq 1.2$$
$$x - 37.5 \leq 1.2$$
$$x \leq 38.7$$

$36.3 \leq x \leq 38.7$

41. Case 1

$$3|x-7| < x - 1$$
$$3(-(x-7)) < x - 1$$
$$3(-x+7) < x - 1$$
$$-3x + 21 < x - 1$$
$$-4x < -22$$
$$x > 5.5$$

Case 2

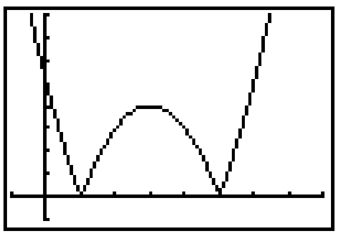
$$3|x-7| < |x-1|$$
$$3(x-7) < x - 1$$
$$3x - 21 < x - 1$$
$$2x < 20$$
$$x < 10$$

$\{x \mid 5.5 < x < 10\}$

42. 30 units^2
The triangular region has vertices $A(0, 10)$, $B(3, 4)$, and $C(8, 14)$. The slope of side AB is -2. The slope of side AC is 0.5, therefore AB is perpendicular to AC. The length of side AB is $3\sqrt{5}$. The length of side AC is $4\sqrt{5}$ the area of the triangle is $0.5(3\sqrt{5})(4\sqrt{5})$ or 30.

43.
$$0.10(90) + 0.15(75) + 0.20(76) + 0.40(80) + 0.15(x) \geq 80$$
$$0.15x \geq 12.55$$
$$x \geq 83\tfrac{2}{3}$$

44.



$[-1, 8]$ scl:1 by $[-1, 8]$ scl:1

44a. $b < 0$

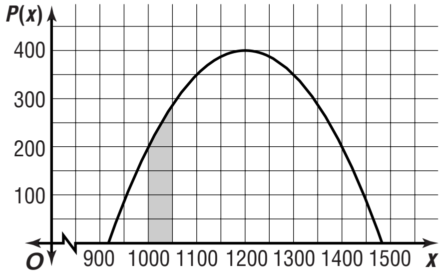
44b. none

44c. $b = 0$ or $b > 4$

44d. $b = 4$

44e. $0 < b < 4$

45a.



45b. The shaded region shows all points (x, y) where x represents the number of cookies sold and y represents the possible profit made for a given week.

46. The graph of $g(x)$ is the graph of $f(x)$ reflected over the x-axis and expanded vertically by a factor of 2.

47. $y = -\frac{1}{a^4} \quad \rightarrow \quad b = -\frac{1}{a^4}$

x-axis: $(-b) = -\frac{1}{a^4}$
$-b = -\frac{1}{a^4}$ no

y-axis: $b = -\frac{1}{(-a)^4}$
$b = -\frac{1}{a^4}$ yes

$y = x$: $(a) = -\frac{1}{(b)^4}$
$a = -\frac{1}{(b)^4}$ no

$y = -x$: $(-a) = -\frac{1}{(-b)^4}$
$-a = -\frac{1}{(b)^4}$ no

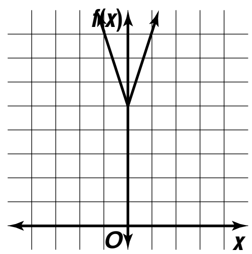
y-axis

48. $$\frac{1}{\begin{vmatrix} 8 & -3 \\ 4 & -5 \end{vmatrix}} \begin{bmatrix} -5 & 3 \\ -4 & 8 \end{bmatrix} = -\frac{1}{28}\begin{bmatrix} -5 & 3 \\ -4 & 8 \end{bmatrix}$$

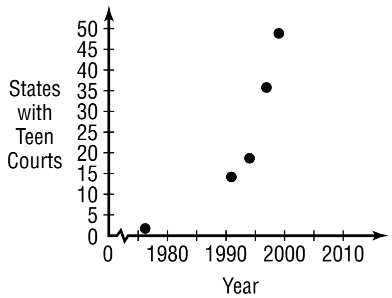
49. $$\frac{3}{4}\begin{bmatrix} 8 & -7 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} \frac{3}{4}(8) & \frac{3}{4}(-7) \\ \frac{3}{4}(-4) & \frac{3}{4}(0) \end{bmatrix}$$
$$= \begin{bmatrix} 6 & \frac{-21}{4} \\ -3 & 0 \end{bmatrix}$$

50.

x	$f(x)$
-2	11
-1	8
0	5
1	8
2	11



51.



52.
$$[f \circ g](4) = f(g(4))$$
$$= f(0.5(4) - 1)$$
$$= f(1)$$
$$= 5(1) + 9$$
$$= 14$$

$$[g \circ f](4) = g(f(4))$$
$$= g(5(4) + 9)$$
$$= g(29)$$
$$= 0.5(29) - 1$$
$$= 13.5$$

53. Student A = 15

Student B = $\frac{1}{3}(15) + 15$ or 20

Let x = number of years past.

$20 - x = 2(15 - x)$

$20 - x = 30 - 2x$

$x = 10$

Page 151 Mid-Chapter Quiz

1. $x^2 + y^2 - 9 = 0 \rightarrow a^2 + b^2 - 9 = 0$

x-axis $\quad a^2 + (-b)^2 - 9 = 0$

$a^2 + b^2 - 9 = 0$ yes

y-axis $\quad (-a)^2 + b^2 - 9 = 0$

$a^2 + b^2 - 9 = 0$ yes

$y = x \quad (b)^2 + (a)^2 - 9 = 0$

$a^2 + b^2 - 9 = 0$ yes

$y = -x \quad (-b)^2 + (-a)^2 - 9 = 0$

$a^2 + b^2 - 9 = 0$ yes

$x^2 + y^2 - 9 = 0 \rightarrow f(x) = \pm\sqrt{-x^2 + 9}$

$f(-x) = \pm\sqrt{-(-x)^2 + 9} \qquad -f(x) = -(\pm\sqrt{-x^2 + 9})$

$f(-x) = \pm\sqrt{-x^2 + 9} \qquad -f(x) = \pm\sqrt{-x^2 + 9}$

yes

x-axis, y-axis, $y = x$, $y = -x$, origin

2. $5x^2 + 6x - 9 = y \rightarrow 5a^2 + 6a - 9 = b$

x-axis $\quad 5a^2 + 6a - 9 = (-b)$

$5a^2 + 6a - 9 = -b$ no

y-axis $\quad 5(-a)^2 + 6(-a) - 9 = b$

$5a^2 - 6a - 9 = b$ no

$y = x \quad 5(b)^2 + 6(b) - 9 = (a)$

$5b^2 + 6b - 9 = a$ no

$y = -x \quad 5(-b)^2 + 6(-b) - 9 = (-a)$

$5b^2 - 6b - 9 = -a$ no

$5x^2 + 6x - 9 = y \rightarrow f(x) = 5x^2 + 6x - 9$

$f(-x) = 5(-x)^2 + 6(-x) - 9$

$= 5x^2 - 6x - 9$

$-f(x) = -(5x^2 + 6x - 9)$

$-f(x) = -5x^2 - 6x + 9$ no

none of these

3. $x = \frac{7}{y} \rightarrow a = \frac{7}{b}$

x-axis $\quad a = \frac{7}{(-b)}$

$a = -\frac{7}{b}$ no

y-axis $\quad (-a) = \frac{7}{b}$

$-a = \frac{7}{b}$ no

$y = x \quad (b) = \frac{7}{(a)}$

$a = \frac{7}{b}$ yes

$y = -x \quad (-b) = \frac{7}{(-a)}$

$a = \frac{7}{b}$ yes

$x = \frac{7}{y} \rightarrow f(x) = \frac{7}{x}$

$f(-x) = \frac{7}{(-x)} \qquad -f(x) = -\left(\frac{7}{x}\right)$

$f(-x) = -\frac{7}{x} \qquad -f(x) = -\frac{7}{x}$ yes

$y = x$, $y = -x$, origin

4. $y = |x| + 1 \rightarrow b = |a| + 1$

x-axis $\quad (-b) = |a| + 1$

$-b = |a| + 1$ no

y-axis $\quad b = |(-a)| + 1$

$b = |a| + 1$ yes

$y = x \quad (a) = |(b)| + 1$

$a = |b| + 1$ no

$y = -x \quad (-a) = |(-b)| + 1$

$-a = |b| + 1$ no

$y = |x| + 1 \rightarrow f(x) = |x| + 1$

$f(-x) = |-x| + 1 \qquad -f(x) = -(|x| + 1)$

$f(-x) = |x| + 1 \qquad -f(x) = -|x| - 1$ no

y-axis

5a. translated down 2 units

5b. reflected over the x-axis, translated right 3 units

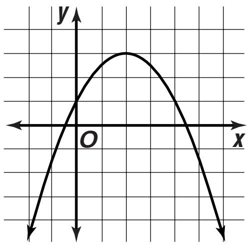
5c. compressed vertically by a factor of $\frac{1}{4}$, translated up 1 unit

6a. expanded vertically by a factor of 3

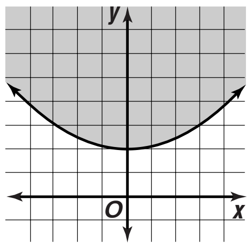
6b. expanded horizontally by a factor of 2 and translated down 1 unit

6c. translated left 1 unit and up 4 units

7.



8.



9. Case 1

$|2x - 7| < 15$

$-(2x - 7) < 15$

$-2x + 7 < 15$

$-2x < 8$

$x > -4$

$-4 < x < 11$

Case 2

$|2x - 7| < 15$

$2x - 7 < 15$

$2x < 22$

$x < 11$

10. $|x - 64| < 3$

Case 1

$|x - 64| < 3$

$-(x - 64) < 3$

$-x + 64 < 3$

$-x < -61$

$x > 61$

$61 < x < 67$

Case 2

$|x - 64| < 3$

$x - 64 < 3$

$x < 67$

3-4 Inverse Functions and Relations

Pages 155–156 Check for Understanding

1. Sample answer: First, let $y = f(x)$. Then interchange x and y. Finally, solve the resulting equation for y.

2. n is odd

3. Sample answer: $f(x) = x^2$

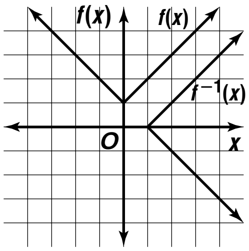
4. Sample answer: If you draw a horizontal line through the graph of the function and it intersects the graph more than once, then the inverse is not a function.

5. She is wrong. The inverse is $f^{-1}(x) = (x - 3)^2 - 2$, which is a function.

6.

$f(x) = \lvert x \rvert + 1$	
x	$f(x)$
−2	3
−1	2
0	1
1	2
2	3

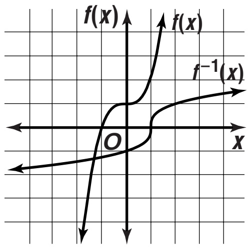
$f^{-1}(x)$	
x	$f^{-1}(x)$
3	−2
2	−1
1	0
2	1
3	2



7.

$f(x) = x^3 + 1$	
x	$f(x)$
−2	−7
−1	0
0	1
1	2
2	9

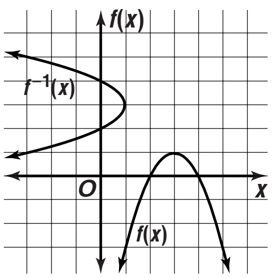
$f^{-1}(x)$	
x	$f^{-1}(x)$
−7	−2
0	−1
1	0
2	1
9	2



8.

$f(x) = -(x - 3)^2 + 1$	
x	$f(x)$
1	−3
2	0
3	1
4	0
5	−3

$f^{-1}(x)$	
x	$f^{-1}(x)$
−3	1
0	2
1	3
0	4
−3	5

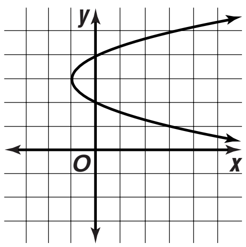


9.
$$f(x) = -3x + 2$$
$$y = -3x + 2$$
$$x = -3y + 2$$
$$x - 2 = -3y$$
$$y = -\tfrac{1}{3}x + \tfrac{2}{3}$$
$f^{-1}(x) = -\frac{1}{3}x + \frac{2}{3}$; $f^{-1}(x)$ is a function.

10.
$$f(x) = \frac{1}{x^3}$$
$$y = \frac{1}{x^3}$$
$$x = \frac{1}{y^3}$$
$$y^3 = \frac{1}{x}$$
$$y = \sqrt[3]{\frac{1}{x}} \text{ or } \frac{1}{\sqrt[3]{x}}$$
$f^{-1}(x) = \frac{1}{\sqrt[3]{x}}$; $f^{-1}(x)$ is a function.

11.
$$f(x) = (x + 2)^2 + 6$$
$$y = (x + 2)^2 + 6$$
$$x = (y + 2)^2 + 6$$
$$x - 6 = (y + 2)^2$$
$$\pm\sqrt{x - 6} = y + 2$$
$$y = -2 \pm \sqrt{x - 6}$$
$f^{-1}(x) = -2 \pm \sqrt{x - 6}$; $f^{-1}(x)$ is not a function.

12. Reflect the graph of $y = x^2$ over the line $y = x$. Then, translate the new graph 1 unit to the left and up 3 units.



13.
$$f(x) = \tfrac{1}{2}x - 5$$
$$y = \tfrac{1}{2}x - 5$$
$$x = \tfrac{1}{2}y - 5$$
$$x + 5 = \tfrac{1}{2}y$$
$$y = 2x + 10$$
$$f^{-1}(x) = 2x + 10$$
$$[f \circ f^{-1}](x) = f(2x + 10) = \tfrac{1}{2}(2x + 10) - 5 = x$$
$$[f^{-1} \circ f](x) = f^{-1}\left(\tfrac{1}{2}x - 5\right) = 2\left(\tfrac{1}{2}x - 5\right) + 10 = x$$
Since $[f \circ f^{-1}](x) = [f^{-1} \circ f](x) = x$, f and f^{-1} are inverse functions.

14a.
$$B(r) = 1000(1 + r)^3$$
$$B = 1000(1 + r)^3$$
$$\frac{B}{1000} = (1 + r)^3$$
$$\sqrt[3]{\frac{B}{1000}} = 1 + r$$
$$r = -1 + \frac{\sqrt[3]{B}}{10}$$

14b. $r = -1 + \frac{\sqrt[3]{B}}{10}$

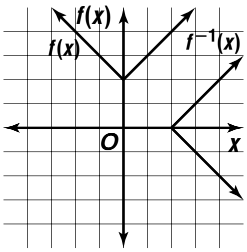
$= -1 + \frac{\sqrt[3]{1100}}{10}$ or 0.0323; 3.23%

Pages 156–158 Exercises

15.

$f(x) = \lvert x \rvert + 2$	
x	$f(x)$
−2	4
−1	3
0	2
1	3
2	4

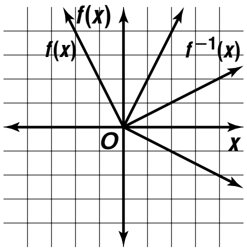
$f^{-1}(x)$	
x	$f^{-1}(x)$
4	−2
3	−1
2	0
3	1
4	2



16.

$f(x) = \lvert 2x \rvert$	
x	$f(x)$
−2	4
−1	2
0	0
1	2
2	4

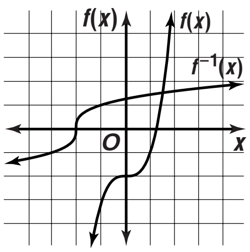
$f^{-1}(x)$	
x	$f^{-1}(x)$
4	−2
2	−1
0	0
2	1
4	2



17.

$f(x) = x^3 - 2$	
x	$f(x)$
−2	−10
−1	−3
0	−2
1	−1
2	6

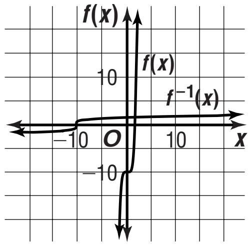
$f^{-1}(x)$	
x	$f^{-1}(x)$
−10	−2
−3	−1
−2	0
−1	1
6	2



18.

$f(x) = x^5 - 10$	
x	$f(x)$
−2	−42
−1	−11
0	−10
1	−9
2	22

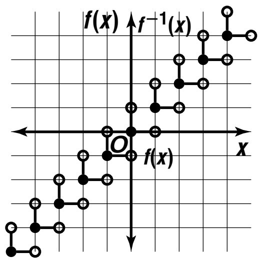
$f^{-1}(x)$	
x	$f^{-1}(x)$
−42	−2
−11	−1
−10	0
−9	1
22	2



19.

$f(x) = [x]$	
x	$f(x)$
$-2 \le x < -1$	−2
$-1 \le x < 0$	−1
$0 \le x < 1$	0
$1 \le x < 2$	1
$2 \le x < 3$	2

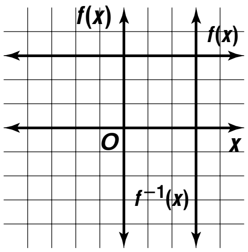
$f^{-1}(x)$	
x	$f^{-1}(x)$
−2	$-2 \le x < -1$
−1	$-1 \le x < 0$
0	$0 < x < 1$
1	$1 \le x < 2$
2	$2 \le x < 3$



20.

$f(x) = 3$	
x	$f(x)$
−2	3
−1	3
0	3
1	3
2	3

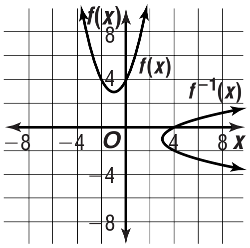
$f^{-1}(x)$	
x	$f^{-1}(x)$
3	−2
3	−1
3	0
3	1
3	2



21.

$f(x) = x^2 + 2x + 4$	
x	$f(x)$
−3	7
−2	4
−1	3
0	4
1	7

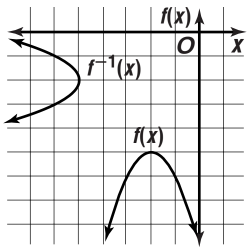
$f^{-1}(x)$	
x	$f^{-1}(x)$
7	−3
4	−2
3	−1
4	0
7	1



22.

$f(x) = -(x + 2)^2 - 5$	
x	$f(x)$
−4	−9
−3	−6
−2	−5
−1	−6
0	−9

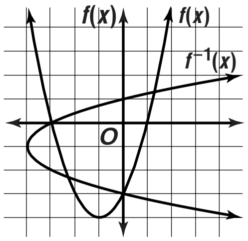
$f^{-1}(x)$	
x	$f^{-1}(x)$
−9	−4
−6	−3
−5	−2
−6	−1
−9	0



23.

$f(x) = (x + 1)^2 - 4$	
x	$f(x)$
−4	5
−2	−3
−1	−4
0	−3
2	5

$f^{-1}(x)$	
x	$f^{-1}(x)$
5	−4
−3	−2
−4	−1
−3	0
5	2



24. $f(x) = x^2 + 4$

$y = x^2 + 4$

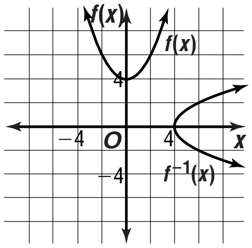
$x = y^2 + 4$

$x - 4 = y^2$

$y = \pm\sqrt{x - 4}$; $f^{-1}(x) = \pm\sqrt{x - 4}$

$f(x) = x^2 + 4$	
x	$f(x)$
−2	8
−1	5
0	4
1	5
2	8

$f^{-1}(x) = \pm\sqrt{x - 4}$	
x	$f^{-1}(x)$
8	±2
5	±1
4	0



25. $f(x) = 2x + 7$

$y = 2x + 7$

$x = 2y + 7$

$x - 7 = 2y$

$y = \frac{x - 7}{2}$

$f^{-1}(x) = \frac{x - 7}{2}$; $f^{-1}(x)$ is a function.

26. $f(x) = -x - 2$

$y = -x - 2$

$x = -y - 2$

$y = -x - 2$

$f^{-1}(x) = -x - 2$; $f^{-1}(x)$ is a function.

27. $f(x) = \frac{1}{x}$

$y = \frac{1}{x}$

$x = \frac{1}{y}$

$y = \frac{1}{x}$

$f^{-1}(x) = \frac{1}{x}$; $f^{-1}(x)$ is a function.

28. $f(x) = -\frac{1}{x^2}$

$y = -\frac{1}{x^2}$

$x = -\frac{1}{y^2}$

$y^2 = -\frac{1}{x}$

$y = \pm\sqrt{-\frac{1}{x}}$

$f^{-1}(x) = \pm\sqrt{-\frac{1}{x}}$; $f^{-1}(x)$ is not a function.

29. $f(x) = (x - 3)^2 + 7$

$y = (x - 3)^2 + 7$

$x = (y - 3)^2 + 7$

$x - 7 = (y - 3)^2$

$\pm\sqrt{x - 7} = y - 3$

$y = 3 \pm \sqrt{x - 7}$

$f^{-1}(x) = 3 \pm \sqrt{x - 7}$; $f^{-1}(x)$ is not a function.

30.
$$f(x) = x^2 - 4x + 3$$
$$y = x^2 - 4x + 3$$
$$x = y^2 - 4y + 3$$
$$x + 1 = y^2 - 4y + 4$$
$$x + 1 = (y - 2)^2$$
$$\pm\sqrt{x + 1} = y - 2$$
$$y = 2 \pm \sqrt{x + 1}$$
$f^{-1}(x) = 2 \pm \sqrt{x + 1}$; $f^{-1}(x)$ is not a function.

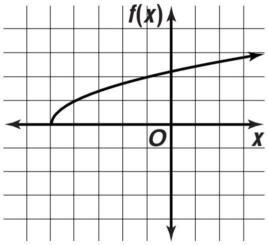
31.
$$f(x) = \frac{1}{x + 2}$$
$$y = \frac{1}{x + 2}$$
$$x = \frac{1}{y + 2}$$
$$y + 2 = \frac{1}{x}$$
$$y = \frac{1}{x} - 2$$
$f^{-1}(x) = \frac{1}{x} - 2$; $f^{-1}(x)$ is not a function.

32.
$$f(x) = \frac{1}{(x - 1)^2}$$
$$y = \frac{1}{(x - 1)^2}$$
$$x = \frac{1}{(y - 1)^2}$$
$$(y - 1)^2 = \frac{1}{x}$$
$$y - 1 = \pm\sqrt{\frac{1}{x}}$$
$$y = 1 \pm \frac{1}{\sqrt{x}}$$
$f^{-1}(x) = 1 \pm \frac{1}{\sqrt{x}}$; $f^{-1}(x)$ is not a function.

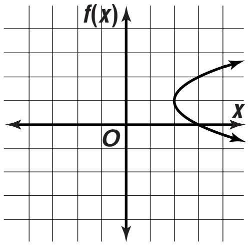
33.
$$f(x) = -\frac{2}{(x - 2)^3}$$
$$y = -\frac{2}{(x - 2)^3}$$
$$x = -\frac{2}{(y - 2)^3}$$
$$(y - 2)^3 = -\frac{2}{x}$$
$$y - 2 = -\sqrt[3]{\frac{2}{x}}$$
$$y = 2 - \sqrt[3]{\frac{2}{x}}$$
$f^{-1}(x) = 2 - \sqrt[3]{\frac{2}{x}}$; $f^{-1}(x)$ is not a function.

34.
$$g(x) = \frac{3}{x^2 + 2x}$$
$$y = \frac{3}{x^2 + 2x}$$
$$x = \frac{3}{y^2 + 2y}$$
$$y^2 + 2y = \frac{3}{x}$$
$$y^2 + 2y + 1 = \frac{3}{x} + 1$$
$$(y + 1)^2 = \frac{3}{x} + 1$$
$$y + 1 = \pm\sqrt{\frac{3}{x} + 1}$$
$$y = -1 \pm \sqrt{\frac{3}{x} + 1}$$
$$g^{-1}(x) = -1 \pm \sqrt{\frac{3}{x} + 1}$$

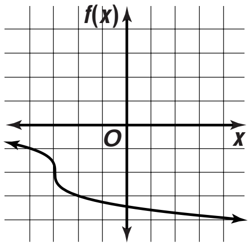
35. Reflect the part of the graph of x^2 that lies in the first quadrant about $y = x$. Then, translate 5 units to the left.



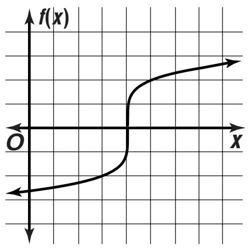
36. Reflect the graph of x^2 about $y = x$. Then, translate 2 units to the right and up 1 unit.



37. Reflect the graph of x^3 about $y = x$ to obtain the graph of $\sqrt[3]{x}$. Reflect the graph of $\sqrt[3]{x}$ about the x-axis. Then, translate 3 units to the left and down 2 units.



38. Reflect the graph of x^5 about $y = x$. Then, translate 4 units to the right. Finally, stretch the translated graph vertically by a factor of 2.



39. $f(x) = -\frac{2}{3}x + \frac{1}{6}$

$y = -\frac{2}{3}x + \frac{1}{6}$

$x = -\frac{2}{3}y + \frac{1}{6}$

$x - \frac{1}{6} = -\frac{2}{3}y$

$y = -\frac{3}{2}x + \frac{1}{4}$

$f^{-1}(x) = -\frac{3}{2}x + \frac{1}{4}$

$[f \circ f^{-1}](x) = f\left(-\frac{3}{2}x + \frac{1}{4}\right)$

$= -\frac{2}{3}\left(-\frac{3}{2}x + \frac{1}{4}\right) + \frac{1}{6}$

$= x - \frac{1}{6} + \frac{1}{6}$

$= x$

$[f^{-1} \circ f](x) = f^{-1}\left(-\frac{2}{3}x + \frac{1}{6}\right)$

$= -\frac{3}{2}\left(-\frac{2}{3}x + \frac{1}{6}\right) + \frac{1}{4}$

$= x - \frac{1}{4} + \frac{1}{4}$

$= x$

Since $[f \circ f^{-1}](x) = [f^{-1} \circ f](x) = x$, f and f^{-1} are inverse functions.

40. $f(x) = (x - 3)^3 + 4$

$y = (x - 3)^3 + 4$

$x = (y - 3)^3 + 4$

$x - 4 = (y - 3)^3$

$\sqrt[3]{x - 4} = y - 3$

$y = 3 + \sqrt[3]{x - 4}$

$f^{-1}(x) = 3 + \sqrt[3]{x - 4}$

$[f \circ f^{-1}](x) = f(3 + \sqrt[3]{x - 4})$

$= [(3 + \sqrt[3]{x - 4}) - 3]^3 + 4$

$= x - 4 + 4$

$= x$

$[f^{-1} \circ f](x) = f^{-1}[(x - 3)^3 + 4]$

$= 3 + \sqrt[3]{[(x - 3)^3 + 4] - 4}$

$= 3 + x - 3$

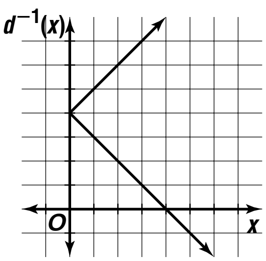
$= x$

Since $[f \circ f^{-1}](x) = [f^{-1} \circ f](x) = x$, f and f^{-1} are inverse functions.

41a.

$d(x) = \lvert x - 4 \rvert$	
x	$d(x)$
6	2
5	1
4	0
3	1
2	2

$d^{-1}(x)$	
x	$d^{-1}(x)$
2	6
1	5
0	4
1	3
2	2



41b. No; the graph of $d(x)$ fails the horizontal line test.

41c. $d^{-1}(x)$ gives the numbers that are 4 units from x on the number line. There are always two such numbers, so d^{-1} associates two values with each x-value. Hence, $d^{-1}(x)$ is not a function.

42a. $v = \sqrt{2gh}$

$v^2 = 2gh$

$h = \frac{v^2}{2g}$

$h = \frac{v^2}{2(32)}$

$h = \frac{v^2}{64}$

42b. $h = \frac{v^2}{64}$

$h = \frac{(75)^2}{64}$

$h = 87.89$

Yes. The pump can propel water to a height of about 88 ft.

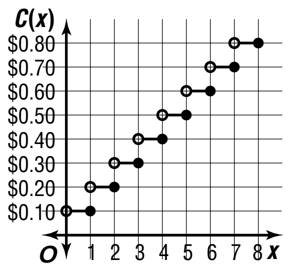
43a. Sample answer: $y = -x$

43b. The graph of the function must be symmetric about the line $y = x$.

43c. Yes, because the line $y = x$ is the axis of symmetry *and* the reflection line.

44a.

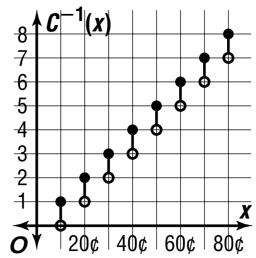
x	$C(x)$
$0 < x \leq 1$	\$0.10
$1 < x \leq 2$	\$0.20
$2 < x \leq 3$	\$0.30
$3 < x \leq 4$	\$0.40
$4 < x \leq 5$	\$0.50



44b. positive real numbers; positive multiples of 10

44c.

x	$C^{-1}(x)$
\$0.10	$0 < x \leq 1$
\$0.20	$1 < x \leq 2$
\$0.30	$2 < x \leq 3$
\$0.40	$3 < x \leq 4$
\$0.50	$4 < x \leq 5$



44d. positive multiples of 10; positive real numbers

44e. $C^{-1}(x)$ gives the possible lengths of phone calls that cost x.

45. It must be translated up 6 units and 5 units to the left; $y = (x - 6)^2 - 5$; $y = 6 \pm \sqrt{x + 5}$.

46a. $KE = \frac{1}{2}mv^2$

$2KE = mv^2$

$\frac{2KE}{m} = v^2$

$v = \pm\sqrt{\frac{2KE}{m}}$

46b. $v = \pm\sqrt{\frac{2KE}{m}}$

$v = \pm\sqrt{\frac{2(15)}{1}}$

$v \approx \pm 5.477$; ± 5.5 m/sec

46c. There are always two velocities.

47a. Yes; if the encoded message is not unique, it may not be decoded properly.

47b. The inverse of the encoding function must be a function so that the encoded message may be decoded.

47c. $C(x) = -2 + \sqrt{x + 3}$

$y = -2 + \sqrt{x + 3}$

$x = -2 + \sqrt{y + 3}$

$x + 2 = \sqrt{y + 3}$

$(x + 2)^2 = y + 3$

$y = (x + 2)^2 - 3$

$C^{-1}(x) = (x + 2)^2 - 3$

47d. $C^{-1}(x) = (x + 2)^2 - 3$
$C^{-1}(1) = (1 + 2)^2 - 3$ or 6, F
$C^{-1}(2.899) = (2.899 + 2)^2 - 3$ or 21, U
$C^{-1}(2.123) = (2.123 + 2)^2 - 3$ or 14, N
$C^{-1}(0.449) = (0.449 + 2)^2 - 3$ or 3, C
$C^{-1}(2.796) = (2.796 + 2)^2 - 3$ or 20, T
$C^{-1}(1.464) = (1.464 + 2)^2 - 3$ or 9, I
$C^{-1}(2.243) = (2.243 + 2)^2 - 3$ or 15, O
$C^{-1}(2.123) = (2.123 + 2)^2 - 3$ or 14, N
$C^{-1}(2.690) = (2.690 + 2)^2 - 3$ or 19, S
$C^{-1}(0) = (0 + 2)^2 - 3$ or 1, A
$C^{-1}(2.583) = (2.583 + 2)^2 - 3$ or 18, R
$C^{-1}(0.828) = (0.828 + 2)^2 - 3$ or 5, E
$C^{-1}(1) = (1 + 2)^2 - 3$ or 6, F
$C^{-1}(2.899) = (2.899 + 2)^2 - 3$ or 21, U
$C^{-1}(2.123) = (2.123 + 2)^2 - 3$ or 14, N
FUNCTIONS ARE FUN

48. Case 1
$|2x + 4| \leq 6$
$-(2x + 4) \leq 6$
$-2x - 4 \leq 6$
$-2x \leq 10$
$x \geq -5$

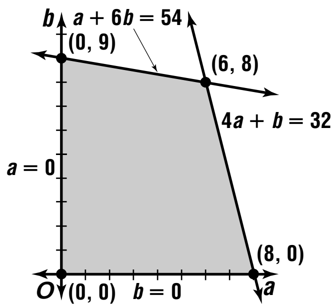
Case 2
$|2x + 4| \leq 6$
$2x + 4 \leq 6$
$2x \leq 2$
$x \leq 1$

$\{x \mid -5 \leq x \leq 1\}$

49. both

50a. $a \geq 0$, $b \geq 0$, $4a + b \leq 32$, $a + 6b \leq 54$

50b.



$G(a, b) = a + b$
$G(0, 0) = 0 + 0$ or 0
$G(0, 9) = 0 + 9$ or 9
$G(6, 8) = 6 + 8$ or 14
$G(8, 0) = 8 + 0$ or 8
14 gallons

51. $4x + 2y = 10$, $y = 6 - x$ $\rightarrow$ $4x + 2y = 10$, $x + y = 6$

$$\begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \end{bmatrix}$$

$$\frac{1}{\begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix}} \begin{bmatrix} 1 & -2 \\ -1 & 4 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & -2 \\ -1 & 4 \end{bmatrix}$$

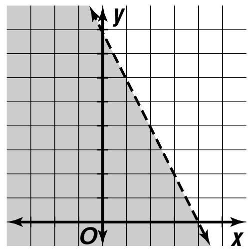
$$\frac{1}{2}\begin{bmatrix} 1 & -2 \\ -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & -2 \\ -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$$

$(-1, 7)$

52. $$\frac{1}{2}\begin{bmatrix} 9 & -3 \\ -6 & 6 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(9) & \frac{1}{2}(-3) \\ \frac{1}{2}(-6) & \frac{1}{2}(6) \end{bmatrix} = \begin{bmatrix} \frac{9}{2} & -\frac{3}{2} \\ -3 & 3 \end{bmatrix}$$

53.



54. $\frac{1}{4} \neq 4$; $\frac{1}{4} \cdot 4 \neq -1$; neither

55. $m = \frac{2 - 7}{5 - 0}$
$= \frac{-5}{5}$ or -1

$y - y_1 = m(x - x_1)$
$y - 7 = -1(x - 0)$
$y = -x + 7$

56. $b + c = 180$
If $\overline{PQ}$ is perpendicular to $\overline{QR}$, then $m\angle PQR = 90$.
Since the angles of a triangle total 180,
$a + d + 90 = 180$.
$a + d = 90$
$a + b + c + d = 180 + 90$ or 270
The correct choice is C.

3-5 Continuity and End Behavior

Page 165 Check for Understanding

1. Sample answer: The function approaches 1 as x approaches 2 from the left, but the function approaches -4 as x approaches 2 from the right. This means the function fails the second condition in the continuity test.

2.

a_n	n	$x \rightarrow$	$p(x) \rightarrow$
positive	even	∞	∞
positive	even	$-\infty$	∞
positive	odd	∞	∞
positive	odd	$-\infty$	$-\infty$

a_n	n	$x \rightarrow$	$p(x) \rightarrow$
negative	even	∞	$-\infty$
negative	even	$-\infty$	$-\infty$
negative	odd	∞	$-\infty$
negative	odd	$-\infty$	∞

3. Infinite discontinuity; $f(x) \rightarrow \infty$ as $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$ as $x \rightarrow \infty$.

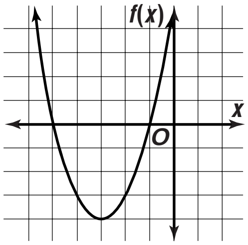
4. $f(x) = x^2$ is decreasing for $x < 0$ and increasing for $x > 0$, $g(x) = -x^2$ is increasing for $x < 0$ and decreasing for $x < 0$. Reflecting a graph switches the monotonicity. In other words, if $f(x)$ is increasing, the reflection will be decreasing and vice versa.

5. No; y is undefined when $x = -3$.

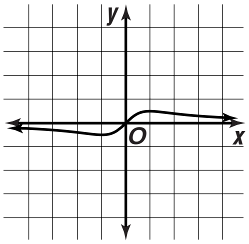
6. No; $f(x)$ approaches 6 as x approaches -2 from the left but $f(x)$ approaches -6 as x approaches -2 from the right.

7. a_n: positive, n: odd
$y \to \infty$ as $x \to \infty$, $y \to -\infty$ as $x \to -\infty$.

8. a_n: negative, n: even
$y \to -\infty$ as $x \to \infty$, $y \to -\infty$ as $x \to -\infty$.

9.


decreasing for $x < -3$; increasing for $x > -3$

10.


decreasing for $x < -1$ and $x > 1$; increasing for $-1 < x < 1$

11a. $t = 4$ **11b.** when $t = 4$ **11c.** 10 amps

Pages 166–168 Exercises

12. Yes; the function is defined when $x = 1$; the function approaches -3 as x approaches 1 from both sides; and $y = -3$ when $x = 1$.

13. No; the function is undefined when $x = 2$.

14. Yes; the function is defined when $x = -3$; the function approaches 0 as x approaches -3 from both sides; and $f(-3) = 0$.

15. Yes; the function is defined when $x = 3$; the function approaches 1 (in fact is equal to 1) as x approaches 3 from both sides; and $y = 1$ when $x = 3$.

16. No; $f(x)$ approaches -7 as x approaches -4 from the left, but $f(x)$ approaches 6 as x approaches -4 from the right.

17. Yes; the function is defined when $x = 1$; $f(x)$ approaches 3 as x approaches 1 from both sides; and $f(1) = 3$.

18. jump discontinuity

19. Sample answer: $x = 0$; $g(x)$ is undefined when $x = 0$.

20. a_n: positive, n: odd
$y \to \infty$ as $x \to \infty$, $y \to -\infty$ as $x \to -\infty$.

21. a_n: negative, n: even
$y \to -\infty$ as $x \to \infty$, $y \to -\infty$ as $x \to -\infty$.

22. a_n: positive, n: even
$y \to \infty$ as $x \to \infty$, $y \to \infty$ as $x \to -\infty$.

23. a_n: positive, n: even
$y \to \infty$ as $x \to \infty$, $y \to \infty$ as $x \to -\infty$.

24.

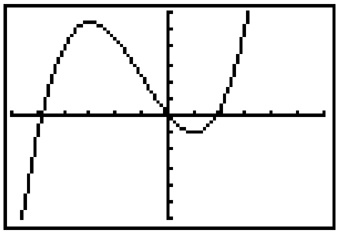
$y = \frac{1}{x^2}$	
x	y
$-10{,}000$	1×10^{-8}
-1000	1×10^{-6}
-100	1×10^{-4}
-10	0.01
0	undefined
10	0.01
100	1×10^{-4}
1000	1×10^{-6}
10,000	1×10^{-8}

$y \to 0$ as $x \to \infty$, $y \to 0$ as $x \to -\infty$.

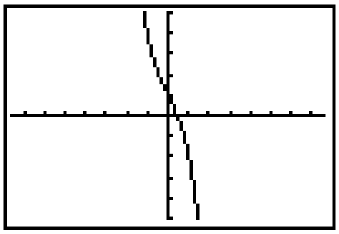
25.

$f(x) = -\frac{1}{x^3} + 2$	
x	$f(x)$
$-10{,}000$	2
-1000	2.000000001
-100	2.000001
-10	2.001
0	undefined
10	1.999
100	1.999999
1000	1.999999999
10,000	2

$f(x) \to 2$ as $x \to \infty$, $f(x) \to 2$ as $x \to -\infty$.

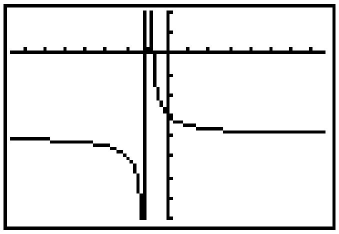
26.


$[-6, 6]$ scl:1 by $[-30, 30]$ scl:5
increasing for $x < -3$ and $x > 1$; decreasing for $-3 < x < 1$

27.


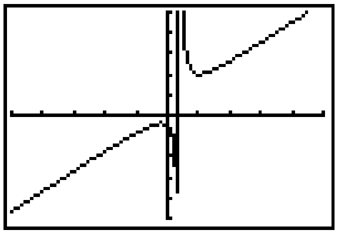
$[-7.6, 7.6]$ scl:1 by $[-5, 5]$ scl:1
decreasing for all x

28.



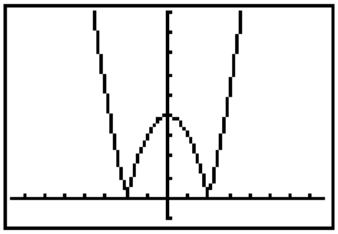
$[-7.6, 7.6]$ scl:1 by $[-8, 2]$ scl:1
decreasing for $x < -1$ and $x > -1$

29.



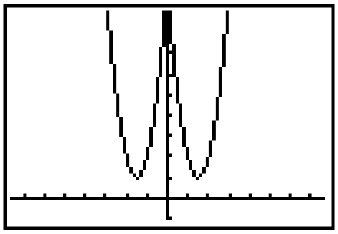
$[-25, 25]$ scl:5 by $[-25, 25]$ scl:5
increasing for $x < -1$ and $x > 5$; decreasing for $-1 < x < 2$ and $2 < x < 5$

30.



$[-7.6, 7.6]$ scl:1 by $[-1, 9]$ scl:1
decreasing for $x < -2$ and $0 < x < 2$; increasing for $-2 < x < 0$ and $x > 2$

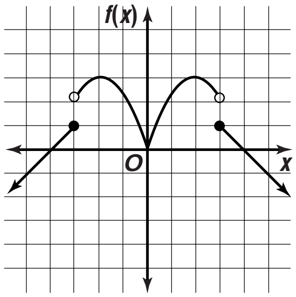
31.



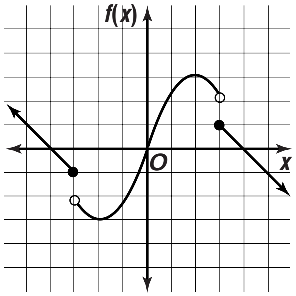
$[-7.6, 7.6]$ scl:1 by $[-1, 9]$ scl:1
decreasing for $x < -\frac{3}{2}$ and $0 < x < \frac{3}{2}$; increasing for $-\frac{3}{2} < x < 0$ and $x > \frac{3}{2}$

32. As the denominator, r, gets larger, the value of $U(r)$ gets smaller. $U(r)$ approaches 0.

33a. Since f is even, its graph must be symmetric with respect to the y-axis. Therefore, f is decreasing for $-2 < x < 0$ and increasing for $x < -2$. f must have a jump discontinuity when $x = -3$ and $f(x) \to -\infty$ as $x \to -\infty$.

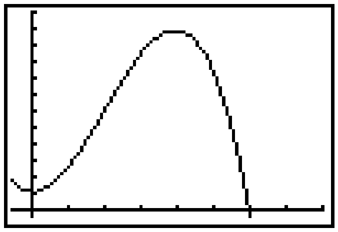


33b. Since f is odd, its graph must be symmetric with respect to the origin. Therefore, f is increasing for $-2 < x < 0$ and decreasing for $x < -2$. f must have a jump discontinuity when $x = -3$ and $f(x) \to \infty$ as $x \to -\infty$.



34a. polynomial

34b.



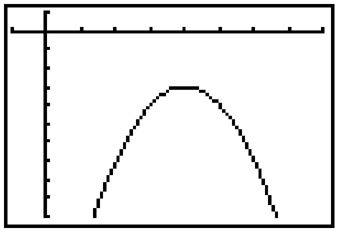
$[-5, 80]$ scl:10 by $[-500, 12000]$ scl:1000
$0.5 < t < 39.5$

34c. $0 < t < 0.5$ and $t > 39.5$

35a. 1954-1956, 1960-1961, 1962-1963, 1966-1968, 1973-1974, 1975-1976, 1977-1978, 1989-1991, 1995-1997

35b. 1956-1960, 1961-1962, 1963-1966, 1968-1973, 1974-1975, 1976-1977, 1978-1989, 1991-1995, 1997-2004

36a.



$[-1, 8]$ scl:1 by $[-10, 1]$ scl:1
$x < 4$

36b. Answers will vary.

36c. The slope is positive. In an interval where a function is increasing, for any two points on the graph, the x- and y-coordinates of one point will be greater than that of the other point, ensuring that the slope of the line through the two points will be positive.

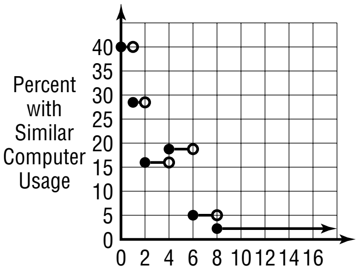
36d. See graph in 36a. $x > 4$

36e. The slope is negative; see students' work.

37a. The function has to be monotonic. If the function were increasing on one interval and decreasing on another interval, the function could not pass the horizontal line test.

37b. The inverse must be monotonic. If the inverse were increasing on one interval and decreasing on another interval, the inverse would fail the horizontal line test. That would mean the function fails the vertical line test, which is impossible.

38a.



38b. $0 < x < 1$, $1 < x < 2$, $2 < x < 4$, $4 < x < 6$, $6 < x < 8$, $x > 8$

39. For the function to be continuous at 2, $bx + a$ and $x^2 + a$ must approach the same value as x approaches 2 from the left and right, respectively. Plugging in $x = 2$ to find that common value gives $2b + a = 4 + a$. Solving for b gives $b = 2$. For the function to be continuous at -2, $\sqrt{-b - x}$ and $bx + a$ must approach the same value as x approaches -2 from the left and right, respectively. Plugging in $x = -2$ gives $\sqrt{-b + 2} = -2b + a$. We already know $b = 2$, so the equation becomes $0 = -4 + a$. Hence, $a = 4$.

40.
$$f(x) = (x + 5)^2$$
$$y = (x + 5)^2$$
$$x = (y + 5)^2$$
$$\pm\sqrt{x} = y + 5$$
$$y = -5 \pm \sqrt{x}$$
$$f^{-1}(x) = -5 \pm \sqrt{x}$$

41. The graph of $g(x)$ is the graph of $f(x)$ translated left 2 units and down 4 units.

42. $f(x, y) = x + 2y$
$f(0, 0) = 0 + 2(0)$ or 0
$f(4, 0) = 4 + 2(0)$ or 4
$f(3, 5) = 3 + 2(5)$ or 13
$f(0, 5) = 0 + 2(5)$ or 10
13, 0

43. $\begin{vmatrix} 5 & -4 \\ 8 & 2 \end{vmatrix} = 5(2) - 8(-4)$ or 42

44a. $c = 47.5h + 35$

44b. $c = 47.5h + 35$
$$c = 47.5\left(2\tfrac{1}{4}\right) + 35$$
$$c = \$141.875$$

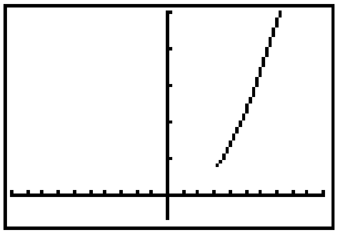
45.
$$f(x) = 2x^2 - 2x + 8$$
$$f(-2) = 2(-2)^2 - 2(-2) + 8$$
$$= 8 + 4 + 8 \text{ or } 20$$

46. The volume of the cube is x^3.
The volume of the other box is $x(x - 1)(x + 1) = x(x^2 - 1)$ or $x^3 - x$.
The difference between the volumes of the two boxes is $x^3 - (x^3 - x)$ or x.
The correct choice is A.

3-5B Gap Discontinuities

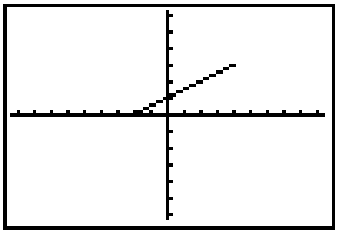
Page 170

1. {all real numbers $x \mid x > 3$}



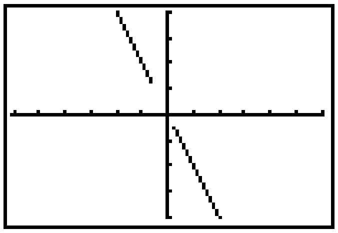
[−10, 10] scl:1 by [−6, 50] scl:10

2. {all real numbers $x \mid -2 \le x \le 4$}



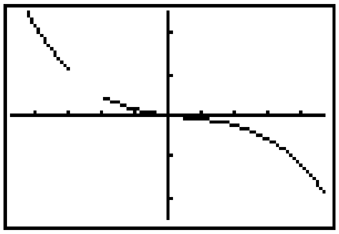
[−9.4, 9.4] scl:1 by [−6.2, 6.2] scl:1

3. {all real numbers $x \mid x < -3$ or $x \ge 1$}



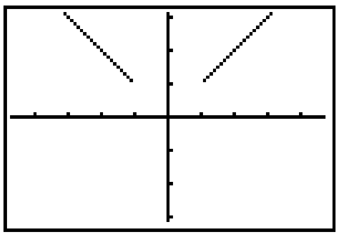
[−18.8, 18.8] scl:1 by [−12.4, 12.4] scl:1

4. {all real numbers $x \mid x \le -3$ or $x > -2$}



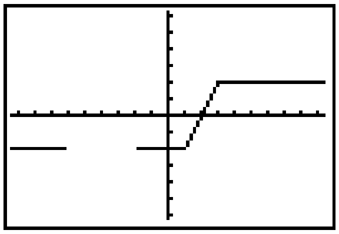
[−4.7, 4.7] scl:1 by [−25, 25] scl:10

5. {all real numbers $x \mid x < -1$ or $x > 1$}



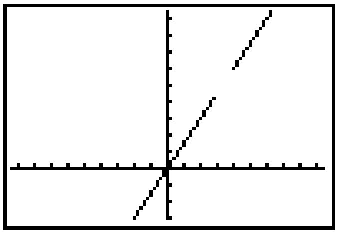
[−4.7, 4.7] scl:1 by [−3.1, 3.1] scl:1

6. {all real numbers $x \mid x < -6$ or $x > -2$}



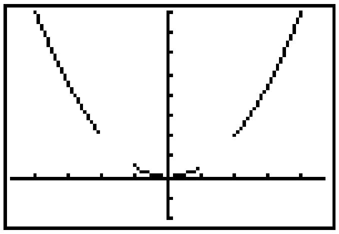
[−9.4, 9.4] scl:1 by [−6.2, 6.2] scl:1

7. {all real numbers $x \mid x < 3$ or $x \geq 4$}



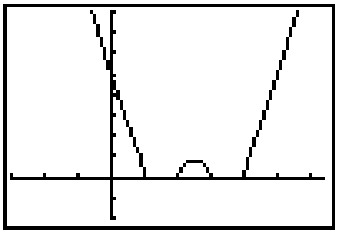
[−9.4, 9.4] scl:1 by [−3, 9.4] scl:1

8. {all real numbers $x \mid x < -2$ or $-1 \leq x < 1$ or $x \geq 2$}



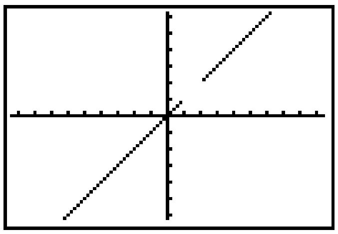
[−4.7, 4.7] scl:1 by [−2, 8] scl:1

9. {all real numbers $x \mid x \leq 1$ or $2 \leq x \leq 3$ or $x \geq 4$}



[−3, 6.4] scl:1 by [−2, 8] scl:1

10. {all real numbers $x \mid x < 1$ or $x > 2$}



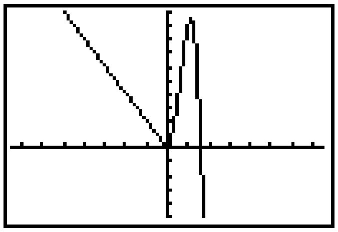
[−9.4, 9.4] scl:1 by [−6.2, 6.2] scl:1

11. Sample answer: $y = \dfrac{x^2}{((x \leq 2) \text{ or } ((x \geq 5) \text{ and } (x \leq 7)) \text{ or } x \geq 8))}$

12. Yes; sample justification: if $f(x)$ is a polynomial function, then the graph of $y = \dfrac{f(x)}{(|x - [[x]]| < 0.25)}$ is like the graph of $f(x)$, but with an infinite number of "interval bites" removed.

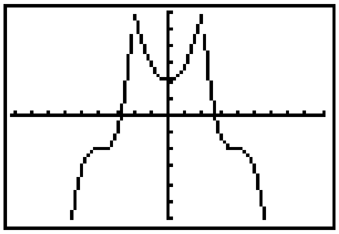
13. Yes; sample justification: the equation $y = \dfrac{x^2(x \leq -2) + (2x - 4)(x \geq 4)}{((x \leq -2) \text{ or } (x \geq 4))}$ is a possible equation for the function described.

14a.



[−15, 15] scl:2 by [−10, 20] scl:2

14b.



[−9.1, 9.1] scl:1 by [−6, 6] scl:1

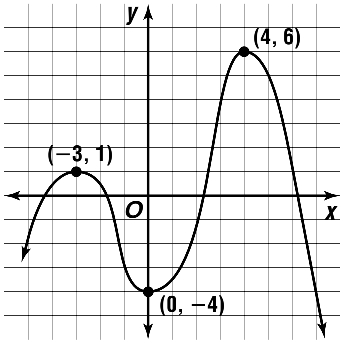
3-6 Critical Points and Extrema

Page 176 Check for Understanding

1. Check values of the function at x-values very close to the critical point. Be sure to check values on both sides. If the function values change from increasing to decreasing, the critical point is a maximum. If the function values change from decreasing to increasing, the critical point is a minimum. If the function values continue to increase or to decrease, the critical point is a point of inflection.

2. rel. min.;
$f(0.99) \approx -3.9997$
$f(1) = -4$
$f(1.01) \approx -3.9997$
By testing points on either side of the critical point, it is evident that the graph of the function is decreasing as x approaches 1 from the left and increasing as x moves away from 1 to the right. Therefore, on the interval $0.99 < x < 1.01$, (1, 4) is a relative minimum.

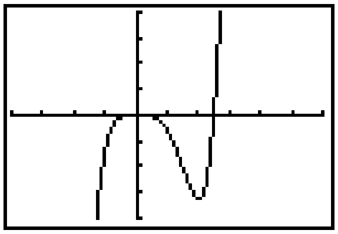
3. Sample answer:



4. rel. min.: (−3, −2); rel. max.: (1, 6)

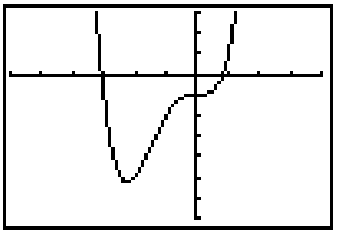
5. rel. min.: (−1, −3); rel. max.: (3, 3)

6. rel. max.: (0, 0); rel. min.: (2, −16)



[−4, 6] scl:1 by [−20, 20] scl:5

7. rel. min.: (−2.25, −10.54)



[−6, 4] scl:1 by [−14, 6] scl:2

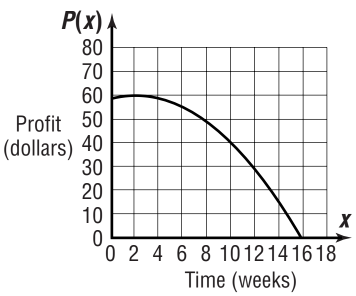
8. $f(-1.1) = 0.907$
$f(-1) = 1$
$f(-0.9) = 0.913$ max.

9. $f(-2.6) = -12.24$
$f(-2.5) = -12.25$
$f(-2.4) = -12.24$ min.

10. $f(-0.1) = -0.00199$
$f(0) = 0$
$f(0.1) = 0.00199$ pt. of inflection

11. $f(-0.1) \approx -0.97$
$f(0) = -1$
$f(0.1) \approx -0.97$ min.

12a. $P(x) = (120 + 10x)(0.48 - 0.03x)$



12b. 2 weeks

12c. $58.80 per acre

12d. Rain or other bad weather could delay harvest and/or destroy part of the crop.

Pages 177–179 Exercises

13. abs. max.: (−4, 1)

14. abs. max.: (−1, 3); rel. min.: (0.5; 0.5); rel. max.: (1.5, 2)

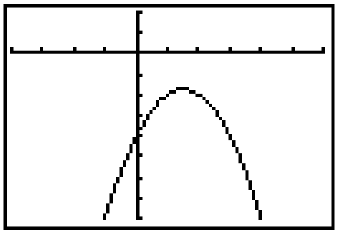
15. rel. max.: (−2, 7): abs. min.: (3, −3)

16. rel. max.: (−6, 4), rel. min.: (−2, −3)

17. abs. min.: (3, −8); rel. max.: (5, −2); rel. min.: (8, −5)

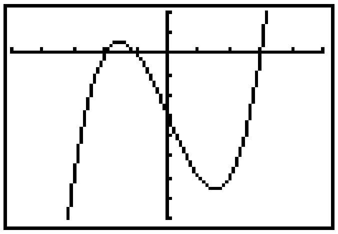
18. no extrema

19. abs. max.: (1.5, −1.75)



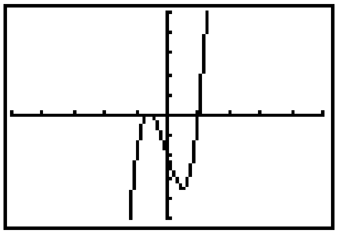
[−5, 5] scl:1 by [−8, 2] scl:1

20. rel. max.: (−1.53, 1.13); rel. min.: (1.53, −13.13)



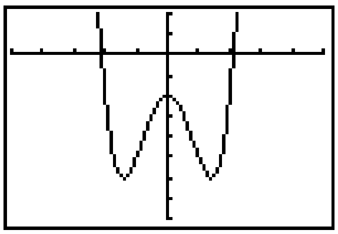
[−5, 5] scl:1 by [−16, 4] scl:2

21. rel. max.: (−0.59, 0.07), rel. min.: (0.47, −3.51)



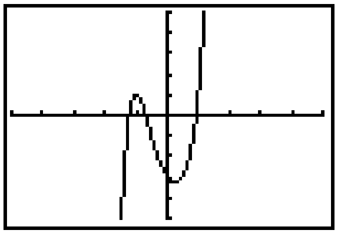
[−5, 5] scl:1 by [−5, 5] scl:1

22. abs. min.: (−1.41, −6), (1.41, −6); rel. max.: (0, −2)



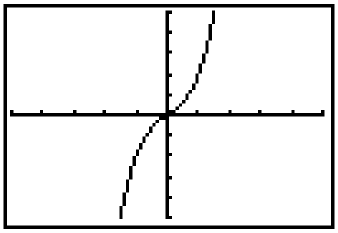
[−5, 5] scl:1 by [−8, 2] scl:1

23. rel. max.: (−1, 1); rel. min.: (0.25, −3.25)



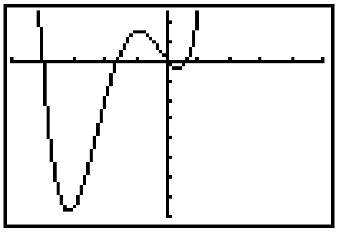
[−5, 5] scl:1 by [−5, 5] scl:1

24. no extrema



[−5, 5] scl:1 by [−5, 5] scl:1

25. abs. min.: (−3.18, −15.47); rel. min. (0.34, −0.80); rel. max.: (−0.91, 3.04)



[−5, 5] scl:1 by [−16, 5] scl:2

26. $f(-0.1) = -0.001$
$f(0) = 0$
$f(0.1) = 0.001$ pt of inflection

27. $f(3.9) = 5.99$
$f(4) = 6$
$f(4.1) = 5.99$ max.

28. $f(-2.6) = -19.48$
$f(-2.5) = -19.5$
$f(-2.4) = -19.48$ min.

29. $f(-0.1) \approx 6.98$
$f(0) = 7$
$f(0.1) \approx 6.98$ max.

30. $f(1.9) \approx -3.96$
$f(2) = -4.82$
$f(2.1) \approx -3.96$ min.

31. $f(2.9) = -0.001$
$f(3) = 0$
$f(3.1) = 0.001$ pt. of inflection

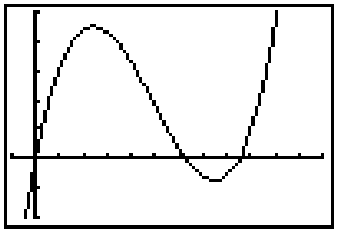
32. $f(-2.1) \approx 4.32$
$f(-2) \approx 4.53$
$f(-1.9) \approx 4.32$ max.

33. $f(0.57) \approx 2.86$
$f\left(\frac{2}{3}\right) \approx 2.85$
$f(0.77) \approx 2.86$ min.

34. The point of inflection is now at $x = -6$ and there is now a minimum at $x = -3$.

35a. $V(x) = 2x(12.5 - 2x)(17 - 2x)$

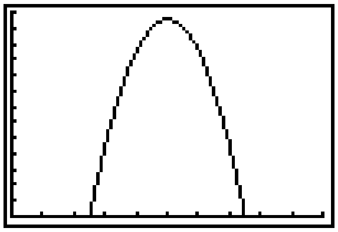
35b.



[−1, 12] scl:1 by [−200, 500] scl:100
2.37 cm by 2.37 cm

35c. See students' work.

36a. $P = sd - 25d$
$= s(-200s + 15{,}000) - 25(-200s + 15{,}000)$
$= -200s^2 + 20{,}000s - 375{,}000$

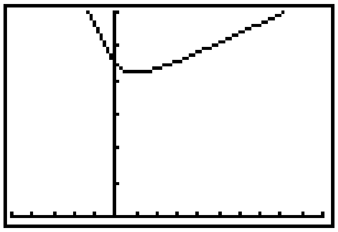


[0, 100] scl:10 by [0, 130,000] scl:10,000
abs. max.: (50, 125,000)
$50

36b. Sample answer: The company's competition might offer a similar product at a lower cost.

37a. $AM^2 = MB^2 + AB^2$
$AM^2 = x^2 + 2^2$
$AM = \sqrt{x^2 + 4}$
$f(x) = 5000(\sqrt{x^2 + 4}) + 3500(10 - x)$

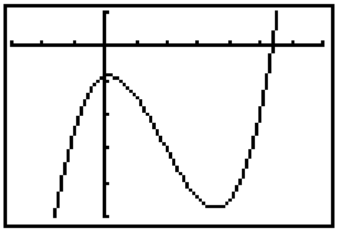
37b.



[−10, 20] scl:2 by [0, 60,000] scl:10,000
abs. min.: (1.96, 42,141.4)
1.96 km from point B

38. equations of the form $y = x^n$ or $y = \sqrt[n]{x}$, where n is odd

39.

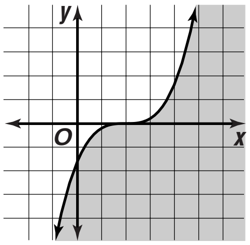


[−3, 7] scl:1 by [−50, 10] scl:10
The particle is at rest when $t \approx 0.14$ and when $t \approx 3.52$. Its positions at these times are $s(0.14) \approx -8.79$ and $s(3.52) \approx -47.51$.

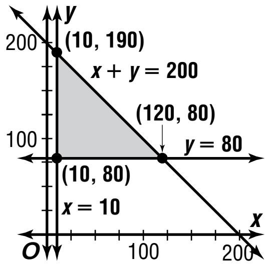
40. If a cubicle has one critical point, then it must be a point of inflection. If it were a relative maximum or minimum, then the end behavior for a cubic would not be satisfied. If a cubic has three critical points, then one must be a maximum, another a minimum, and the third a point of inflection.

41. No; the function is undefined when $x = 5$.

42.

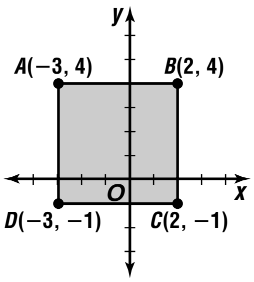


43. Let x = units of notebook paper.
Let y = units of newsprint.
$x + y \le 200$
$x \ge 10$
$y \ge 80$



$P(x, y) = 400x + 350y$
$P(10, 80) = 400(10) + 350(80)$ or 32,000
$P(10, 190) = 400(10) + 350(190)$ or 70,500
$P(120, 80) = 400(120) + 350(80)$ or 76,000
120 units of notebook and 80 units of newsprint

44.



$-3 \le x \le 2,\ -1 \le y \le 4$

45. $\begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix} = 1(5) - 2(3)$ or -1; yes

46.
$$3A = 3\begin{bmatrix} 4 & -2 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 3(4) & 3(-2) \\ 3(5) & 3(7) \end{bmatrix} \text{ or } \begin{bmatrix} 12 & -6 \\ 15 & 21 \end{bmatrix}$$
$$2B = 2\begin{bmatrix} -3 & 5 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 2(-3) & 2(5) \\ 2(-4) & 2(3) \end{bmatrix} \text{ or } \begin{bmatrix} -6 & 10 \\ -8 & 6 \end{bmatrix}$$
$$3A + 2B = \begin{bmatrix} 12 & -6 \\ 15 & 21 \end{bmatrix} + \begin{bmatrix} -6 & 10 \\ -8 & 6 \end{bmatrix} = \begin{bmatrix} 12 + (-6) & -6 + 10 \\ 15 + (-8) & 21 + 6 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 7 & 27 \end{bmatrix}$$

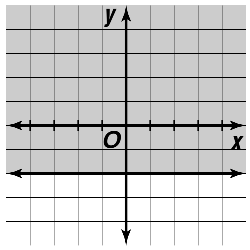
47. Let x = number of 1-point free throws.
Let y = number of 2-point field goals.
Let z = number of 3-point field goals.
$1x + 2y + 3z = 32$
$x + y + z = 17$
$y = 0.50(18)$

$$\begin{array}{l} 1x + 2y + 3z = 32 \\ -1(x + y + z = 17) \end{array} \rightarrow \begin{array}{r} 1x + 2y + 3z = 32 \\ -x - y - z = -17 \\ \hline y + 2z = 15 \end{array}$$

$y = 0.50(18)$ $\qquad y + 2z = 15$
$y = 9$ $\qquad 9 + 2z = 15$
$\qquad z = 3$

$x + y + z = 17$
$x + 9 + 3 = 17$
$x = 5$ $\qquad$ 5 free throws, 9 2-point field goals, 3 3-point field goals

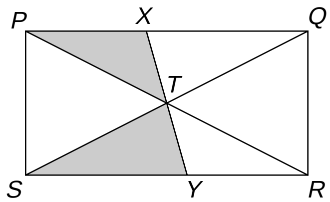
48. $y + 6 \ge 4$
$y \ge -2$



49. $2x + 3y = 15 \rightarrow y = -\frac{2}{3}x + 5$
$6x = 4y + 16 \rightarrow y = \frac{3}{2}x - 4$
$-\frac{2}{3} \cdot \frac{3}{2} = -1$; perpendicular

50. A relation relates members of a set called the domain to members of a set called the range. In a function, the relation must be such that each member of the domain is related to one and only one member of the range. You can use the vertical line test to determine whether a graph is the graph of a function.

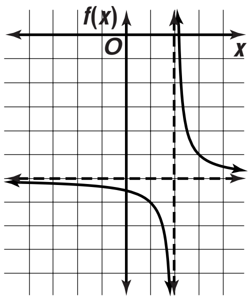
51. The area of $\triangle PTX$ is equal to the area of $\triangle RTY$. The area of $\triangle STR$ is 25% of the area of rectangle $PQRS$. The correct choice is D.



3-7 Graphs of Rational Functions

Pages 185–186 Check for Understanding

1.

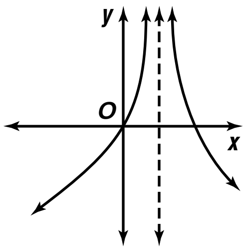


1a. $x = 2, y = -6$

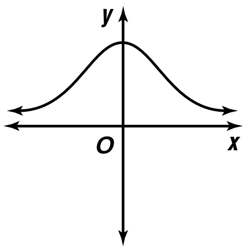
1b. $y = \frac{1}{x-2} - 6$

2. Sample graphs:

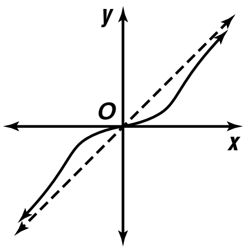
Vertical Asymptote



Horizontal Asymptote



Slant Asymptote



3. Sample answer: $f(x) = \frac{x(x+1)}{x+1}$

4. False; sample explanation: if that x-value also causes the numerator to be 0, there could be a hole instead of a vertical asymptote.

5. $x = 5$

$$f(x) = \frac{x}{x-5}$$
$$y = \frac{x}{x-5}$$
$$y(x-5) = x$$
$$xy - 5y = x$$
$$xy - x = 5y$$
$$x(y-1) = 5y$$
$$x = \frac{5y}{y-1};\ y = 1$$

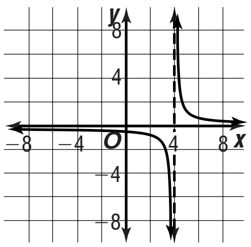
6. $x = 2, x = -1$

$$y = \frac{x^3}{(x-2)(x+1)}$$
$$y = \frac{x^3}{x^2 - x - 2}$$
$$y = \frac{\frac{x^3}{x^3}}{\frac{x^2}{x^3} - \frac{x}{x^3} - \frac{2}{x^3}}$$
$$y = \frac{1}{\frac{1}{x} - \frac{1}{x^2} - \frac{2}{x^3}}$$

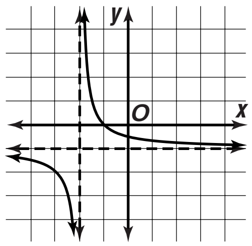
no horizontal asymptotes

7. $f(x) = \frac{1}{x+1} - 2$

8. The parent graph is translated 4 units right. The vertical asymptote is now at $x = 4$. The horizontal asymptote, $y = 0$, is unchanged.



9. The parent graph is translated 2 units left and down 1 unit. The vertical asymptote is now at $x = -2$ and the horizontal asymptote is now $y = -1$.

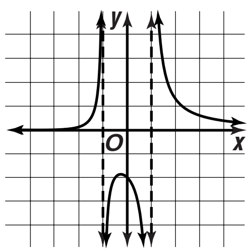


10.

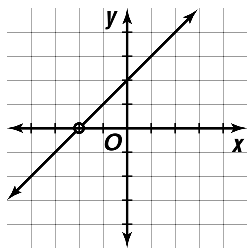
$$\begin{array}{r} 3x + 5 \\ x - 3\overline{)3x^2 - 4x + 5} \\ \underline{3x^2 - 9x} \\ 5x + 5 \\ \underline{5x - 15} \\ 20 \end{array} \rightarrow 3x + 5 + \frac{20}{x-3}$$

$y = 3x + 5$

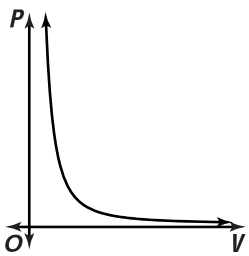
11.



12.



13a.



13b. $P = 0, V = 0$

13c. The pressure approaches 0.

Pages 186–188 Exercises

14. $x = -4$

$$f(x) = \frac{2x}{x+4}$$
$$y = \frac{2x}{x+4}$$
$$y(x+4) = 2x$$
$$xy + 4y = 2x$$
$$xy - 2x = -4y$$
$$x(y-2) = -4y$$
$$x = \frac{-4y}{y-2};\ y = 2$$

15. $x = -6$

$$y = \frac{x^2}{x+6}$$
$$y = \frac{\frac{x^2}{x^2}}{\frac{x^2}{x^2} + \frac{6}{x^2}}$$
$$y = \frac{1}{\frac{1}{x} + \frac{6}{x^2}}$$

no horizontal asymptote

16. $x = -\frac{1}{2}, x = 5$

$$y = \frac{x-1}{(2x+1)(x-5)}$$
$$y = \frac{x-1}{2x^2 - 9x - 5}$$
$$y = \frac{\frac{x}{x^2} - \frac{1}{x^2}}{\frac{2x^2}{x^2} - \frac{9x}{x^2} - \frac{5}{x^2}}$$
$$y = \frac{\frac{1}{x} - \frac{1}{x^2}}{2 - \frac{9}{x} - \frac{5}{x^2}};\ y = 0$$

17. $x = -1, x = -3$

$$y = \frac{x-2}{x^2 - 4x + 3}$$
$$y = \frac{\frac{x}{x^2} + \frac{2}{x^2}}{\frac{x^2}{x^2} + \frac{4x}{x^2} + \frac{3}{x^2}}$$
$$y = \frac{\frac{1}{x} + \frac{2}{x^2}}{1 + \frac{4}{x} + \frac{3}{x^2}};\ y = 0$$

18. no vertical asymptote,

$$y = \frac{x^2}{x^2+1}$$
$$y = \frac{\frac{x^2}{x^2}}{\frac{x^2}{x^2} + \frac{1}{x^2}}$$
$$y = \frac{1}{1 + \frac{1}{x^2}};\ y = 1$$

19. $x = 1$

$$y = \frac{(x+1)^2}{x^2 - 1}$$
$$y = \frac{x^2 + 2x + 1}{x^2 - 1}$$
$$y = \frac{\frac{x^2}{x^2} + \frac{2x}{x^2} + \frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}}$$
$$y = \frac{1 + \frac{2}{x} + \frac{1}{x^2}}{1 - \frac{1}{x^2}};\ y = 1$$

20. $x = 2$

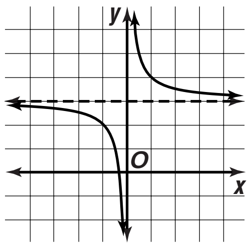
$$y = \frac{x^3}{(x-2)^4}$$
$$y = \frac{x^3}{x^4 - 8x^3 + 24x^2 - 32x + 16}$$
$$y = \frac{\frac{x^3}{x^4}}{\frac{x^4}{x^4} - \frac{8x^3}{x^4} + \frac{24x^2}{x^4} - \frac{32x}{x^4} + \frac{16}{x^4}}$$
$$y = \frac{\frac{1}{x}}{1 - \frac{8}{x} + \frac{24}{x^2} - \frac{32}{x^3} + \frac{16}{x^4}};\ y = 0$$

21. $f(x) = \frac{1}{x+3} + 1$

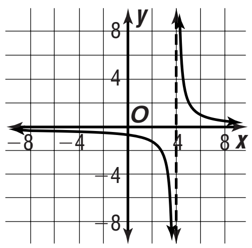
22. $f(x) = \frac{1}{x-2} - 3$

23. $f(x) = -\frac{1}{x} + 1$

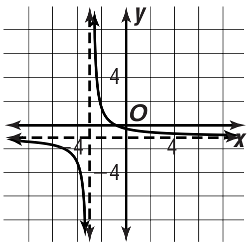
24. The parent graph is translated 3 units up. The vertical asymptote, $x = 0$, is unchanged. The horizontal asymptote is now $y = 3$.



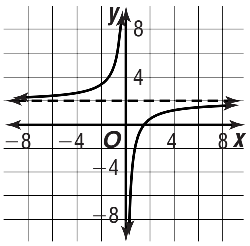
25. The parent graph is translated 4 units right and expanded vertically by a factor of 2. The vertical asymptote is now $x = 4$. The horizontal asymptote, $y = 0$, is unchanged.



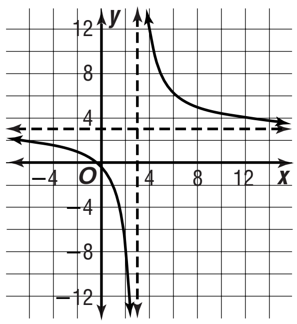
26. The parent graph is translated 3 units left. The translated graph is then expanded vertically by a factor of 2 and translated 1 unit down. The vertical asymptote is now $x = -3$ and the horizontal asymptote is now $y = -1$.



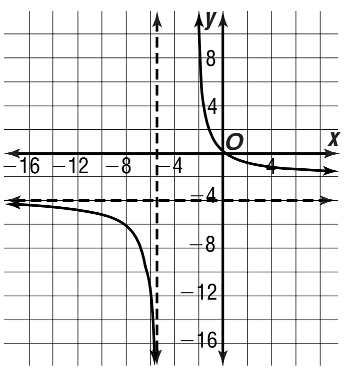
27. The parent graph is expanded vertically by a factor of 3, reflected about the x-axis, and translated 2 units up. The vertical asymptote, $x = 0$, is unchanged. The horizontal asymptote is now $y = 2$.



28. The parent graph is translated 3 units right. The translated graph is expanded vertically by a factor of 10 and then translated 3 units up. The vertical asymptote is $x = 3$ and the horizontal asymptote is $y = 3$.



29. The parent graph is translated 5 units left. The translated graph is expanded vertically by a factor of 22 and then translated 4 units down. The vertical asymptote is $x = -5$ and the horizontal asymptote is $y = -4$.



30.

$$\begin{array}{r} x - 1 \\ x + 4\overline{)x^2 + 3x - 3} \\ \underline{x^2 + 4x} \\ -x - 3 \\ \underline{-x - 4} \\ 1 \end{array} \quad \rightarrow \quad x - 1 + \frac{1}{x + 4} \qquad y = x - 1$$

31.

$$\begin{array}{r} x + 3 \\ x\overline{)x^2 + 3x - 4} \\ \underline{x^2 \qquad\quad} \\ 3x - 4 \\ \underline{3x \quad\;} \\ -4 \end{array} \quad \rightarrow \quad x + 3 - \frac{4}{x} \qquad y = x + 3$$

32.

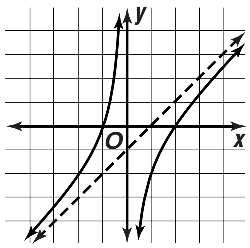
$$\begin{array}{r} x - 2 \\ x^2 + 1\overline{)x^3 - 2x^2 + x - 4} \\ \underline{x^3 \qquad\quad + x \quad} \\ -2x^2 \qquad - 4 \\ \underline{-2x^2 \qquad - 2} \\ -2 \end{array} \quad \rightarrow \quad x - 2 - \frac{2}{x^2 + 1} \qquad y = x - 2$$

33.

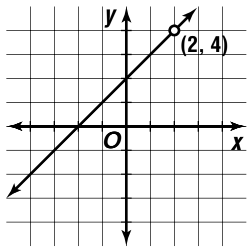
$$\begin{array}{r} \frac{1}{2}x - \frac{5}{4} \\ 2x - 3\overline{)x^2 - 4x + 1} \\ \underline{x^2 - \frac{3}{2}x \qquad} \\ -\frac{5}{2}x + 1 \\ \underline{-\frac{5}{2}x + \frac{15}{4}} \\ -\frac{11}{4} \end{array} \quad \rightarrow \quad \frac{1}{2}x - \frac{5}{4} - \frac{\frac{11}{4}}{2x - 3} \qquad y = \frac{1}{2}x - \frac{5}{4}$$

34. No; the degree of the numerator is 2 more than that of the denominator.

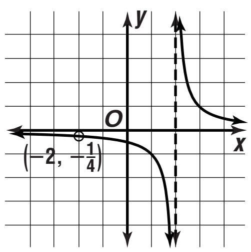
35.



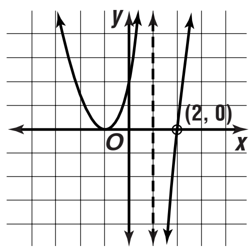
36.



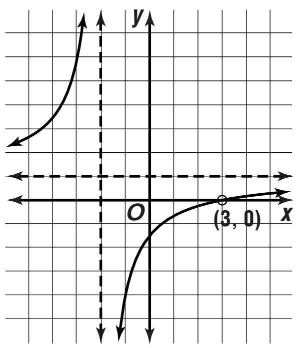
37.



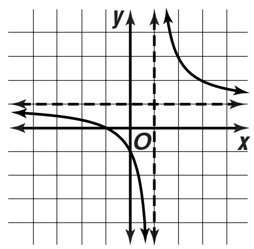
38.



39.



40.



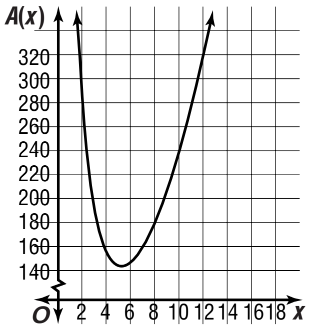
41a. $C(t) = \frac{480 + 3t}{40 + t}$

41b.
$$C(t) = \frac{480 + 3t}{40 + t}$$
$$10 = \frac{480 + 3t}{40 + t}$$
$$400 + 10t = 480 + 3t$$
$$7t = 80$$
$$t \approx 11.43 \text{ L}$$

42. Sample answer: The circuit melts or one of the components burns up.

43. To get the proper x-intercepts, $x - 2$ and $x + 3$ should be factors of the numerator. The vertical asymptote indicates that $x - 4$ should be a factor of the denominator. To get point discontinuity at $(-5, 0)$, make $x + 5$ a factor of both the numerator and denominator with a bigger exponent in the numerator. Thus, a sample answer is $f(x) = \frac{(x - 2)(x + 3)(x + 5)^2}{(x - 4)(x + 5)}$.

44a.
$$V = x^2 \cdot h \qquad A(x) = 4x \cdot h + 2x^2$$
$$120 = x^2 \cdot h \qquad A(x) = 4x\left(\frac{120}{x^2}\right) + 2x^2$$
$$\frac{120}{x^2} = h \qquad A(x) = \frac{480}{x} + 2x^2$$

44b.


44c. The surface area approaches infinity.

45. If the degree of the denominator is larger than that of the numerator, then $y = 0$ will be a horizontal asymptote. To make the graph intersect the x-axis, the simplest numerator to use is x. Thus, a sample answer is $f(x) = \frac{x}{x^2 + 1}$.

46a. A vertical asymptote at $r = 0$ and a horizontal asymptote at $F = 0$.

46b. The force of repulsion increases without bound as the charges are moved closer and closer together. The force of repulsion approaches 0 as the charges are moved farther and farther apart.

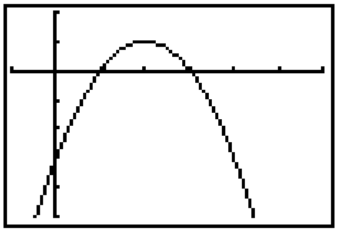
47a. $\frac{a^2 - 9}{a - 3}$

47b.

x	2.9	2.99	3	3.01	3.1
m	5.9	5.99	—	6.01	6.1

The slope approaches 6.

48. abs. max.: (2, 1)



[−1, 6] scl:1 by [−5, 2] scl:1

49.
$$x^2 - 9 = y$$
$$y^2 - 9 = x$$
$$y^2 = x + 9$$
$$y = \pm\sqrt{x + 9}$$

50. $f(x, y) = y - x$
$f(0, 0) = 0 - 0$ or 0
$f(4, 0) = 0 - 4$ or -4
$f(3, 5) = 5 - 3$ or 2
$f(0, 5) = 5 - 0$ or 5
5; -4

51.
$$-4\begin{bmatrix} -6 & 5 \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} -4(-6) & -4(5) \\ -4(8) & -4(-4) \end{bmatrix} = \begin{bmatrix} 24 & -20 \\ -32 & 16 \end{bmatrix}$$

52. Let x = price of film and y = price of sunscreen.
$$8x + 2y = 35.10$$
$$3x + y = 14.30 \qquad 8x + 2y = 35.10$$
$$y = 14.30 - 3x \qquad 8x + 2(14.30 - 3x) = 35.10$$
$$2x + 28.60 = 35.10$$
$$x = 3.25$$
$$y = 14.30 - 3x$$
$$y = 14.30 - 3(3.25)$$
$$y = 4.55$$
\$3.25; \$4.55

53.
$$x + y \geq 3 \qquad x + y \geq 3$$
$$0 + 0 \overset{?}{\geq} 3 \qquad 3 + 2 \overset{?}{\geq} 3$$
$$0 \ngeq 3 \text{ no} \qquad 5 \geq 3 \text{ yes}$$
$$x + y \geq 3 \qquad x + y \geq 3$$
$$-4 + 2 \overset{?}{\geq} 3 \qquad -2 + 4 \overset{?}{\geq} 3$$
$$-2 \ngeq 3 \text{ no} \qquad 2 \ngeq 3 \text{ no}$$
(3, 2)

54. $15y - x = 1 \rightarrow y = \frac{1}{15}x + \frac{1}{15}$

55.
$$[f \circ g](x) = f(g(x)) = f(2 - x^2) = 8(2 - x^2) = 16 - 8x^2$$
$$[g \circ f](x) = g(f(x)) = g(8x) = 2 - (8x)^2 = 2 - 64x^2$$

56. Let x = the width of each card and y = the height of each card. The rectangle has a base of $4x$ or $5y$. The rectangle has a height of $x + y$.

$A = bh$

$180 = 4x(x + y)$ $\quad$ $4x = 5y$

$180 = 4x\left(x + \frac{4x}{5}\right)$ $\quad$ $y = \frac{4x}{5}$

$180 = \frac{36x^2}{5}$ $\quad$ $y = \frac{4(5)}{5}$

$25 = x^2$ $\quad$ $y = 4$

$5 = x$

Perimeter $= 2(4x) + 2(x + y)$

$P = 2(4 \cdot 5) + 2(5 + 4)$

$P = 58$ in.

The correct choice is B.

3-8 Direct, Inverse, and Joint Variation

Pages 193–194 Check for Understanding

1a. inverse

1b. neither

1c. direct

2. Sample answer:
Suppose y varies directly as x^n.
Then $y_1 = kx_1{}^n$ and $y_2 = kx_2{}^n$

$y_1 = kx_1{}^n$

$\frac{y_1}{y_2} = \frac{kx_1}{kx_2}$ *Division property of equality.*

$\frac{y_1}{y_2} = \frac{x_1}{x_2}$ *Simplify.*

3. The line does not go through the origin, therefore its equation is not of the form $y = kx^n$.

4a. Sample answer: The amount of money earned varies directly with the number of hours worked.

4b. Sample answer: The distance traveled by a car varies inversely as the amount of gas in the car.

4c. Sample answer: The volume of a cylinder varies jointly as its height and the radius of its base.

5. $xy = k$ $\quad$ $xy = 12$

$4(3) = k$ $\quad$ $15y = 12$

$12 = k$ $\quad$ $y = \frac{4}{5}$

6. $y = kx^2$ $\quad$ $y = -\frac{2}{3}x^2$

$-54 = k(9)^2$ $\quad$ $y = -\frac{2}{3}(6)^2$

$-\frac{2}{3} = k$ $\quad$ $y = -24$

7. $y = kxz^3$ $\quad$ $y = 0.5xz^3$

$16 = k(4)(2)^3$ $\quad$ $y = (0.5)(-8)(-3)^3$

$0.5 = k$ $\quad$ $y = 108$

8. $y = \frac{kxz}{w^2}$ $\quad$ $y = \frac{0.4xz}{w^2}$

$3 = \frac{k(3)(10)}{2^2}$ $\quad$ $y = \frac{0.4(4)(20)}{4^2}$

$0.4 = k$ $\quad$ $y = 2$

9. y varies directly as x^4; $\frac{1}{7}$.

10. A varies jointly as ℓ and w; 1

11. y varies inversely as x; -3.

12a. $V = khg^2$

$288 = k(40)(1.5)^2$

$3.2 = k$

$V = 3.2hg^2$

12b. $V = 3.2hg^2$

$V = 3.2(75)(2)^2$

$V = 960$

$50 \cdot 960 = 48{,}000 \text{ m}^3$

Pages 194–196 Exercises

13. $y = kx$ $\quad$ $y = 0.2x$

$0.3 = k(1.5)$ $\quad$ $y = 0.2(6)$

$0.2 = k$ $\quad$ $y = 1.2$

14. $xy = k$ $\quad$ $xy = -50$

$25(-2) = k$ $\quad$ $x(-40) = -50$

$-50 = k$ $\quad$ $x = 1.25$

15. $y = kxz$ $\quad$ $y = 15xz$

$36 = k(1.2)(2)$ $\quad$ $y = 15(0.4)(3)$

$15 = k$ $\quad$ $y = 18$

16. $x^2y = k$ $\quad$ $x^2y = 36$

$(2)^2(9) = k$ $\quad$ $3^2y = 36$

$36 = k$ $\quad$ $y = 4$

17. $r = kt^2$ $\quad$ $r = 16t^2$

$4 = k\left(\frac{1}{2}\right)^2$ $\quad$ $r = 16\left(\frac{1}{4}\right)^2$

$16 = k$ $\quad$ $r = 1$

18. $\sqrt{x}y = k$

$\sqrt{1.21}(0.44) = k$

$0.484 = k$

$\sqrt{x}y = 0.484$ or $y = 0.484 \cdot \frac{1}{\sqrt{x}}$

$y = 0.484 \cdot \frac{1}{\sqrt{0.16}}$

$y = 1.21$

19. $y = kx^3z^2$ $\quad$ $y = \frac{1}{12}x^3z^2$

$-9 = k(-3)^3(2)^2$ $\quad$ $y = \frac{1}{12}(-4)^3(-3)^2$

$\frac{1}{12} = k$ $\quad$ $y = -48$

20. $y = \frac{kx}{z^2}$ $\quad$ $y = \frac{0.3x}{z^2}$

$\frac{1}{6} = \frac{k(20)}{6^2}$ $\quad$ $y = \frac{0.3(14)}{5^2}$

$0.3 = k$ $\quad$ $y = 0.168$

21. $y = \frac{kxz}{w}$ $\quad$ $y = \frac{2xz}{w}$

$-3 = \frac{k(2)(-3)}{4}$ $\quad$ $y = \frac{2(4)(-7)}{-4}$

$2 = k$ $\quad$ $y = 14$

22. $y = \frac{kz^2}{x^3}$ $\quad$ $y = \frac{-2z^2}{x^2}$

$-6 = \frac{k(9)^2}{3^3}$ $\quad$ $y = \frac{-2(-4)^2}{6^3}$

$-2 = k$ $\quad$ $y = -\frac{4}{27}$

23. $a = \frac{kb^2}{c}$ $\quad$ $a = \frac{15b^2}{c}$

$45 = \frac{k(6)^2}{12}$ $\quad$ $96 = \frac{15b^2}{10}$

$15 = k$ $\quad$ $\pm 8 = b$

24. $x^2y = k$ $\quad$ $yx^2 = 32$

$(4)^2(2) = k$ $\quad$ $8x^2 = 32$

$32 = k$ $\quad$ $x = \pm 2$

25. C varies directly as d; π.

26. y varies directly as x; $\frac{1}{4}$.

27. y varies jointly as x and the square of z; $\frac{4}{3}$.

28. V varies directly as the cube of r; $\frac{4}{3}\pi$.

29. y varies inversely as the square of x; $\frac{5}{4}$.

30. y varies inversely as the square root of x; 2.

31. A varies jointly as h and the quantity $b_1 + b_2$; 0.5.

32. y varies directly as x and inversely as the square of z; $\frac{1}{3}$.

33. y varies directly as x^2 and inversely as the cube of z; 7.

34. y varies jointly as the product of the cube of x and z and inversely as the square of w.

35a. Joint variation; to reduce torque one must either reduce the distance or reduce the mass on the end of the fulcrum. Thus, torque varies directly as the mass and the distance from the fulcrum. Since there is more than one quantity in direct variation with the torque on the seesaw, the variation is joint.

35b. $T_1 = km_1d_1$ and $T_2 = km_2d_2$

$T_1 = T_2$

$km_1d_1 = km_2d_2$ *Substitution property of equality*

$m_1d_1 = m_2d_2$

35c. $m_1d_1 = m_2d_2$

$75(3.3) = (125)d_2$

$1.98 = d_2$; 1.98 meters

36a. $tr = k$

36b. $tr = k$ $\quad$ $tr = 36{,}000$

$45(800) = k$ $\quad$ $t(1000) = 36{,}000$

$36{,}000 = k$ $\quad$ $t = 36$ minutes

37. If y varies directly as x then there is a nonzero constant k such that $y = kx$. Solving for x, we find $x = \frac{1}{k}y \cdot \frac{1}{k}$ is a nonzero constant, so x varies directly as y.

38a. $I = \frac{k}{d^2}$

38b. $I = \frac{k}{d^2}$ $\quad$ $a^2 + b^2 = c^2$ $\quad$ $I = \frac{576}{d^2}$

$16 = \frac{k}{6^2}$ $\quad$ $(6)^2 + (25)^2 = c^2$ $\quad$ $I = \frac{576}{(6.5)^2}$

$576 = k$ $\quad$ $6.5 = c$ $\quad$ $I \approx 13.6$ lux

39. a is doubled

$a = \frac{kb^2}{c^3}$

$a = \frac{k\left(\frac{1}{2}b\right)^2}{\left(\frac{1}{2}c\right)^3}$

$a = \frac{\frac{1}{4}kb^2}{\frac{1}{8}c^3}$

$a = 2\frac{kb^2}{c^3}$

40a. $F = G\frac{m_1 \cdot m_2}{d^2}$

40b. $F = G\frac{m_1 \cdot m_2}{d^2}$

$1.99 \times 10^{20} = G\frac{(5.98 \times 10^{24})(7.36 \times 10^{22})}{(3.84 \times 10^8)^2}$

$6.67 \times 10^{-11} = G$; $6.67 \times 10^{-11}\ \frac{\text{N} \cdot m^2}{\text{kg}^2}$

40c. $F = G\frac{m_1 \cdot m_2}{d^2}$

$= (6.67 \times 10^{-11})\ \frac{(5.98 \times 10^{24})(1.99 \times 10^{30})}{(1.50 \times 10^{11})^2}$

$\approx 3.53 \times 10^{22}$ N

40d. $3.53 \times 10^{22} = (1.99 \times 10^{20})x$

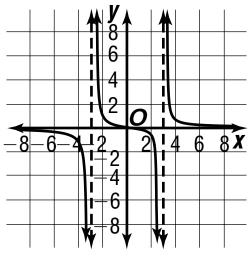
$178 \approx x$; about 178 times greater

41. $R = \frac{kL}{\pi r^2}$ $\quad$ $R = \frac{1.68 \times 10^{-8}(3)}{\pi(0.003)^2}$

$1.07 \times 10^{-2} = \frac{k \cdot 2}{\pi(0.001)^2}$ $\quad$ $R = 1.78 \times 10^{-3}\ \Omega$

$1.68 \times 10^{-8} \approx k$

42.



43. $f(x) = (x - 3)^3 + 6$

$y = (x - 3)^3 + 6$

$x = (y - 3)^3 + 6$

$x - 6 = (y - 3)^3$

$\sqrt[3]{x - 6} = y - 3$

$y = \sqrt[3]{x - 6} + 3$

$f^{-1}(x) = \sqrt[3]{x - 6} + 3$; $f^{-1}(x)$ is a function.

44. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & -1 & -3 \\ 2 & -2 & -4 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -3 & 1 & 3 \\ 2 & -2 & -4 & 0 \end{bmatrix}$

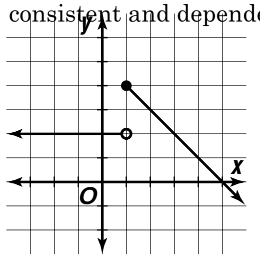
$A'(-1, 2)$, $B'(-$
$-2)$, $C'(1, -4)$, $D'(3, 0)$

45. $4x - 2y = 7 \quad \rightarrow \quad y = 2x - \frac{7}{2}$

$-12x + 6y = -21 \quad \rightarrow \quad y = 2x - \frac{7}{2}$

consistent and dependent

46.



47. $m = \frac{18.6 - 23.2}{2000 - 1995}$ $\quad$ $y - 18.6 = -0.92(x - 2000)$

$= \frac{-4.6}{5}$ or -0.92 $\quad$ $y = -0.92x + 1858.60$

48. $144 = 4^2 \cdot 9$ or $12^2 \cdot 1$

12 is divisible by 3, 4, 6, and 12.

The correct choice is D.

Chapter 3 Study Guide and Assessment

Page 197 Understanding and Using the Vocabulary

1. even **2.** continuous **3.** point
4. decreasing **5.** maximum **6.** rational
7. inverse **8.** monotonic **9.** slant
10. Joint

Pages 198–200 Skills and Concepts

11. $f(-x) = -2(-x)$ $\quad -f(x) = -(-2x)$
$f(-x) = 2x$ $\quad -f(x) = 2x$ yes

12. $f(-x) = (-x)^2 + 2$ $\quad -f(x) = -(x^2 + 2)$
$f(-x) = x^2 + 2$ $\quad -f(x) = -x^2 - 2$ no

13. $f(-x) = (-x)^2 - (-x) + 3$
$f(-x) = x^2 + x + 3$
$-f(x) = -(x^2 - x + 3)$
$-f(x) = -x^2 + x - 3$ no

14. $f(-x) = (-x)^3 - 6(-x) + 1$
$f(-x) = -x^3 + 6x + 1$
$-f(x) = -(x^3 - 6x + 1)$
$-f(x) = -x^3 + 6x - 1$ no

15. $xy = 4 \quad \rightarrow \quad ab = 4$
x-axis $\quad a(-b) = 4$
$-ab = 4$ no
y-axis $\quad (-a)b = 4$
$-ab = 4$ no
$y = x$ $\quad (b)(a) = 4$
$ab = 4$ yes
$y = -x$ $\quad (-b)(-a) = 4$
$ab = 4$ yes
$y = x$ and $y = -x$

16. $x + y^2 = 4 \quad \rightarrow \quad a + b^2 = 4$
x-axis $\quad a + (-b)^2 = 4$
$a + b^2 = 4$ yes
y-axis $\quad (-a) + b^2 = 4$
$-a + b^2 = 4$ no
$y = x$ $\quad (b) + (a)^2 = 4$
$a^2 + b = 4$ no
$y = -x$ $\quad (-b) + (-a)^2 = 4$
$a^2 - b = 4$ no
x-axis

17. $x = -2y \quad \rightarrow \quad a = -2b$
x-axis $\quad a = -2(-b)$
$a = 2b$ no
y-axis $\quad (-a) = -2b$
$a = 2b$ no
$y = x$ $\quad (b) = -2(a)$
$b = -2a$ no
$y = -x$ $\quad (-b) = -2(-a)$
$b = -2a$ no; none

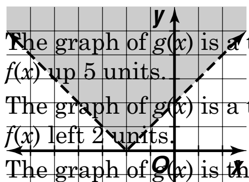
18. $x^2 = \frac{1}{y} \quad \rightarrow \quad a^2 = \frac{1}{b}$
x-axis $\quad a^2 = \frac{1}{(-b)}$
$a^2 = -\frac{1}{b}$ no
y-axis $\quad (-a)^2 = \frac{1}{b}$
$a^2 = \frac{1}{b}$ yes
$y = x$ $\quad (b)^2 = \frac{1}{(a)}$
$b^2 = \frac{1}{a}$ no
$y = -x$ $\quad (-b)^2 = \frac{1}{(-a)}$
$b^2 = -\frac{1}{a}$ no; y-axis

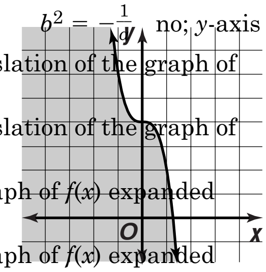
19. The graph of $g(x)$ is a translation of the graph of $f(x)$ up 5 units.

20. The graph of $g(x)$ is a translation of the graph of $f(x)$ left 2 units.

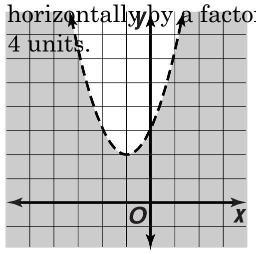
21. The graph of $g(x)$ is the graph of $f(x)$ expanded vertically by a factor of 6.

22. The graph of $g(x)$ is the graph of $f(x)$ expanded horizontally by a factor of $\frac{4}{3}$ and translated down 4 units.

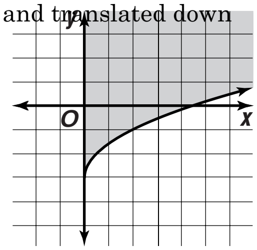




23.



24.



25.

26.

27. Case 1
$|4x + 5| > 7$
$-(4x + 5) > 7$
$-4x - 5 > 7$

Case 2
$|4x + 5| > 7$
$4x + 5 > 7$
$4x > 2$

$-4x > 12$
$x < -3$
$x > 0.5$
$\{x \mid x < -3 \text{ or } x > 0.5\}$

28. Case 1
$|x - 3| + 2 \leq 11$
$-(x - 3) + 2 \leq 11$
$-x + 5 \leq 11$
$-x \leq 6$
$x \geq -6$

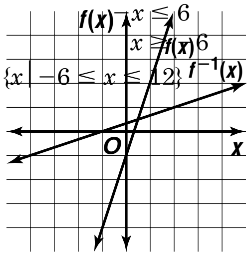
Case 2
$|x - 3| + 2 \leq 11$
$x - 3 + 2 \leq 11$
$x \leq 12$

$\{x \mid -6 \leq x \leq 12\}$

29.

$f(x) = 3x - 1$	
x	$f(x)$
−2	−7
−1	−4
0	−1
1	2

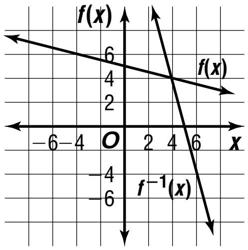
$f^{-1}(x)$	
x	$f^{-1}(x)$
−7	−2
−4	−1
−1	0
2	1



30.

$f(x) = -\frac{1}{4}x + 5$	
x	$f(x)$
−2	5.5
−1	5.25
0	5
1	4.75
2	4.5

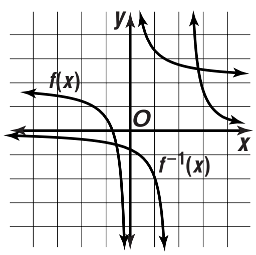
$f^{-1}(x)$	
x	$f^{-1}(x)$
5.5	−2
5.25	−1
5	0
4.75	1
4.5	2



31.

$f(x) = \frac{2}{x} + 3$	
x	$f(x)$
−3	2.3
−2	2
−1	1
$-\frac{1}{2}$	−1
0	—
$\frac{1}{2}$	7
1	5
2	4
3	3.7

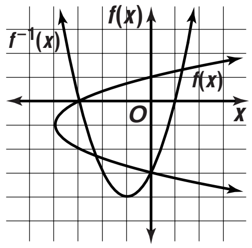
$f^{-1}(x)$	
x	$f^{-1}(x)$
2.3	−3
2	−2
1	−1
−1	$-\frac{1}{2}$
7	$\frac{1}{2}$
5	1
4	2
3.7	3



32.

$f(x) = (x + 1)^2 - 4$	
x	$f(x)$
−3	0
−2	−3
−1	−4
0	−3
1	0

$f^{-1}(x)$	
x	$f^{-1}(x)$
0	−3
−3	−2
−4	−1
−3	0
0	1



33.
$$f(x) = (x - 2)^3 - 8$$
$$y = (x - 2)^3 - 8$$
$$x = (y - 2)^3 - 8$$
$$x + 8 = (y - 2)^3$$
$$\sqrt[3]{x + 8} = y - 2$$
$$y = \sqrt[3]{x + 8} + 2$$
$$f^{-1}(x) = \sqrt[3]{x + 8} + 2; \text{ yes}$$

34.
$$f(x) = 3(x + 7)^4$$
$$y = 3(x + 7)^4$$
$$x = 3(y + 7)^4$$
$$\frac{x}{3} = (y + 7)^4$$
$$\pm\sqrt[4]{\frac{x}{3}} = y + 7$$
$$y = -7 \pm \sqrt[4]{\frac{x}{3}}$$
$$f^{-1}(x) = -7 \pm \sqrt[4]{\frac{x}{3}}; \text{ no}$$

35. Yes; the function is defined when $x = 2$; the function approaches 6 as x approaches 2 from both sides; and $y = 6$ when $x = 2$.

36. No; the function is undefined when $x = -1$.

37. Yes; the function is defined when $x = 1$; the function approaches 2 as x approaches 1 from both sides; and $y = 2$ when $x = 1$.

38. a_n: negative, n: odd
$y \to -\infty$ as $x \to \infty$, $y \to \infty$ as $x \to -\infty$.

39. a_n: positive, n: odd
$y \to \infty$ as $x \to \infty$, $y \to -\infty$ as $x \to -\infty$.

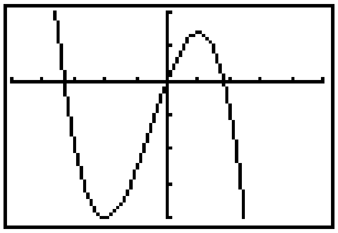
40.

$y = \frac{1}{x^2} + 1$	
x	y
−1000	1.000001
−100	1.0001
−10	1.01
1	2
10	1.01
100	1.0001
1000	1.000001

$y \to 1$ as $x \to \infty$, $y \to 1$ as $x \to -\infty$.

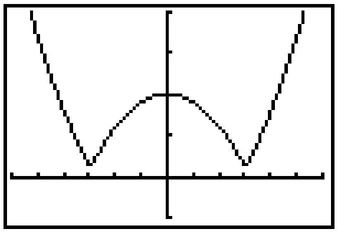
41. a_n: positive, n: odd
$y \to \infty$ as $x \to \infty$, $y \to -\infty$ as $x \to -\infty$.

42.



$[-5, 5]$ scl:1 by $[-20, 10]$ scl:5
decreasing for $x < -2$ and $x > 1$;
increasing for $-2 < x < 1$

43.



$[-6, 6]$ scl:1 by $[-5, 20]$ scl:5
decreasing for $x < -3$ and $0 < x < 3$;
increasing for $-3 < x < 0$ and $x > 3$

44. abs. max.: $(-2, 1)$

45. rel. max.: $(0, 4)$, rel. min.: $(2, 0)$

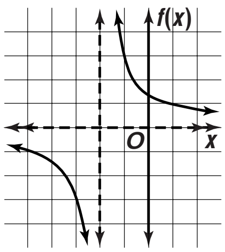
46. $f(2.9) = 0.029$
$f(3) = 0$
$f(3.1) = 0.031$ min.

47. $f(-0.1) = 6.996$
$f(0) = 7$
$f(0.1) = 7.004$ pt of inflection

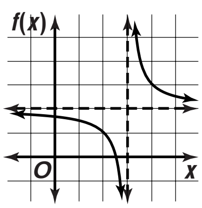
48. $f(x) = \frac{1}{x} + 1$

49. $f(x) = -\frac{2}{x}$

50. The parent graph is translated 2 units left and expanded vertically by a factor of 3. The vertical asymptote is now $x = -3$. The horizontal asymptote, $f(x) = 0$, is unchanged.



51. The parent graph is translated 3 units right and then translated 2 units up. The vertical asymptote is now $x = 3$ and the horizontal asymptote is $f(x) = 2$.



52. $x = 1$

$$y = \frac{x}{x-1}$$
$$y = \frac{\frac{x}{x}}{\frac{x}{x} - \frac{1}{x}}$$
$$y = \frac{1}{1 - \frac{1}{x}}; \quad y = 1$$

53. $x = -2$

$$y = \frac{x^2+1}{x+2}$$
$$y = \frac{\frac{x^2}{x^2} + \frac{1}{x^2}}{\frac{x}{x^2} + \frac{2}{x^2}}$$
$$y = \frac{1 + \frac{1}{x^2}}{\frac{1}{x} + \frac{2}{x^2}}$$

no horizontal asymptotes

54. $x = -3,$

$$y = \frac{(x-3)^2}{x^2-9}$$
$$y = \frac{x^2 - 6x + 9}{x^2 - 9}$$
$$y = \frac{\frac{x^2}{x^2} - \frac{6x}{x^2} + \frac{9}{x^2}}{\frac{x^2}{x^2} - \frac{9}{x^2}}$$
$$y = \frac{1 - \frac{6}{x} + \frac{9}{x^2}}{1 - \frac{9}{x^2}}; \; y = 1$$

55.

$$\begin{array}{r} x + 2 \\ x\overline{)x^2 + 2x + 1} \\ \underline{x^2 } \\ 2x \\ \underline{2x } \\ 1 \end{array} \quad \rightarrow \quad x + 2 + \frac{1}{x}$$

yes; $y = x + 2$

56.
$y = kxz$
$5 = k(-4)(-2)$
$0.625 = k$

$y = 0.625xz$
$y = 0.625(-6)(-3)$
$y = 11.25$

57.
$y = \frac{k}{\sqrt{x}}$
$20 = \frac{k}{\sqrt{49}}$
$140 = k$

$y = \frac{140}{\sqrt{x}}$
$10 = \frac{140}{\sqrt{x}}$
$\sqrt{x} = 14$
$x = 196$

58.
$y = \frac{kx^2}{z}$
$7.2 = \frac{k(0.3)^2}{4}$
$320 = k$

$y = \frac{320x^2}{z}$
$y = \frac{320(1)^2}{40}$
$y = 8$

Page 201 Applications and Problem Solving

59. $|x - 6.5| \le 0.2$;

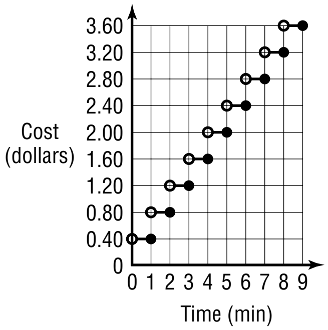
Case 1
$|x - 6.5| \le 0.2$
$-(x - 6.5) \le 0.2$
$-x + 6.5 \le 0.2$
$-x \le -6.3$
$x \ge 6.3$

Case 2
$|x - 6.5| \le 0.2$
$x - 6.5 \le 0.2$
$x \le 6.7$

$6.3 \le x \le 6.7$

60a.

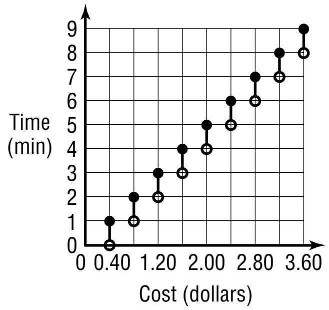
x	$C(x)$
$0 < x \le 1$	0.40
$1 < x \le 2$	0.80
$2 < x \le 3$	1.20
$3 < x \le 4$	1.60
$4 < x \le 5$	2.00
$5 < x \le 6$	2.40



60b. positive real numbers; positive multiples of \$0.40

60c.

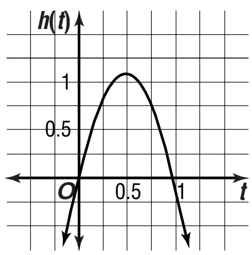
x	$C^{-1}(x)$
0.40	$0 < x \le 1$
0.80	$1 < x \le 2$
1.20	$2 < x \le 3$
1.60	$3 < x \le 4$
2.00	$4 < x \le 5$
2.40	$5 < x \le 6$



60d. positive multiples of \$0.40; positive real numbers

60e. $C^{-1}(x)$ gives the possible number of minutes spent using the scanner that cost x dollars.

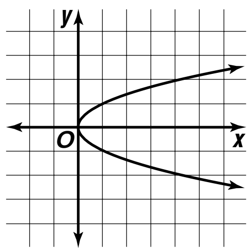
61a.



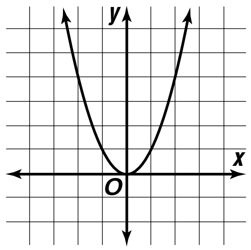
61b. 1.08 m

Page 201 Open-Ended Assessment

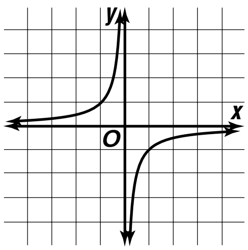
1a. Sample answer: $x = y^2$



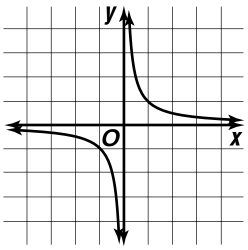
1b. Sample answer: $y = x^2$



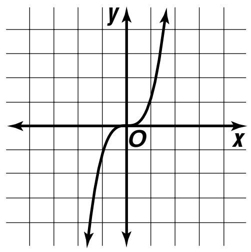
1c. Sample answer: $-xy = 1$



1d. Sample answer: $xy = 1$

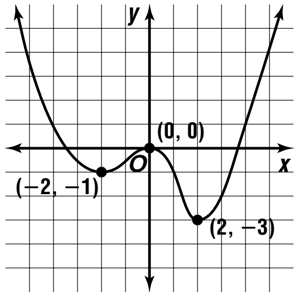


1e. Sample answer: $y = x^3$



2. Sample answer: $-2(x - 4)^2 + 1$

3a. Sample answer:



3b. abs. min.: (2, −3); rel. max.: (0, 0); rel. min.: (−2, −1)

Chapter 3 SAT & ACT Preparation

Page 203 SAT & ACT Practice

1. Always factor or simplify algebraic expressions when possible. Notice that the numerator in the problem is the difference of two squares, $a^2 - b^2$. Factor it.

$\frac{y^2 - 9}{3y - 9} = \frac{(y + 3)(y - 3)}{3y - 9}$

Factor the denominator. Both the numerator and denominator contain the factor $(y - 3)$. Simplify the fraction.

$\frac{(y + 3)(y - 3)}{3y - 9} = \frac{(y + 3)(y - 3)}{3(y - 3)} = \frac{y + 3}{3}$

The correct answer is E.

2. You need to find the statement that is *not* true. Compare the given information with each answer choice. Choice A looks like $x + y = z$, except for the numbers. Multiply both sides of the equation $x + y = z$ by 2.

$2(x + y) = 2z$ or $2x + 2y = 2z$

So choice A is true. For choice B, start with $x = y$ and subtract y from each side.

$x - y = y - y = 0$

So choice B is true. For choice C, start with $x = y$ and subtract z from each side.

$x - z = y - z$

So choice C is true. For choice D, substitute y for x and $x + y$ for z.

$x = \frac{z}{2}$

$y = \frac{x + y}{2} = \frac{2y}{2}y$

So choice D is also true. For choice E, write each side of the equation in terms of y.

$z - y = (x + y) - x = y$

$2x = 2y$

$y \neq 2y$

So choice E is *not* true. The correct choice is E.

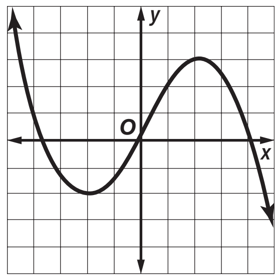
3. Notice that 450 miles is the distance to Grandmother's house, not the round trip. This is a multiple-step problem. First calculate the number of gallons of gasoline used in each direction of the trip.

$\frac{\text{miles}}{\text{miles per gallon}} = \text{gallons}$

$\frac{450}{25} = 18 \text{ gallons}$

On the trip to Grandmother's, the cost of gasoline is 18 gallons × \$1.25 per gallon or \$22.50.
On the trip back, the gasoline cost is 18 gallons × \$1.50 per gallon or \$27.00. The difference between the costs is \$4.50.
A faster way to find the cost difference is to reason that each gallon cost \$0.25 more on the trip back. So the total amount more that was paid was 18 gallons × \$0.25 or \$4.50. The correct choice is B.

4.



The portion of the graph of $f(x)$ which is shown crosses the x-axis 3 times.
The correct choice is D.

5. Notice that the denominators are all powers of ten. Carefully convert each fraction to a decimal. Then add the three decimals.

$\frac{900}{10} + \frac{90}{100} + \frac{9}{1000} = 90 + 0.9 + 0.009 = 90.909$

The correct choice is C. You could also use your calculator on this problem.

6. Combine like terms.

$(10x^4 - x^2 + 2x - 8) - (3x^4 + 3x^3 + 2x + 9)$
$= (10x^4 - 3x^4) + (-3x^3) + (-x^2) + (2x - 2x) + (-8 - 9)$
$= 7x^4 - 3x^3 - x^2 + 0 - 17$

The correct choice is A.

7. One method of solving this problem is to "plug in" a number in place of n. Choose a number that when divided by 8, has a remainder of 5. For example, choose 21.

$21 = 2(8) + 5$

Then use this value for n in the answer choices. Find the expression that has a remainder of 7.

Choice A: $\frac{n + 1}{8} = \frac{21 + 1}{8} = \frac{22}{8} = 2R6$

The remainder is 6.

Choice B: $\frac{n + 2}{8} = \frac{21 + 2}{8} = \frac{23}{8} = 2R7$

You could also reason that since n divided by 8 has a remainder of 5, then $(n + 2)$ divided by 8 will have a remainder of $(5 + 2)$ or 7. The correct choice is B.

8. Simplify the expression inside the square root symbol. Factor 100 from each term. Then factor the trinomial.

$$\frac{\sqrt{100x^2 + 600x + 900}}{x + 3} = \frac{\sqrt{100(x^2 + 6x + 9)}}{x + 3}$$
$$= \frac{10\sqrt{x^2 + 6x + 9}}{x + 3}$$
$$= \frac{10\sqrt{(x + 3)(x + 3)}}{x + 3}$$
$$= \frac{10\sqrt{(x + 3)^2}}{x + 3}$$
$$= \frac{10(x + 3)}{x + 3}$$
$$= 10$$

The correct choice is B.

9. Since $a + b = c$, substitute $a + b$ for c in $a - c = 5$. So, $a - (a + b) = 5$. Then $-b = 5$ or $b = -5$. Substitute -5 for b in $b - c = 3$. So, $-5 - c = 3$. Then $-c = 8$ or $c = -8$.
The correct choice is B.

10. There are two equations and two variables, so this is a system of equations. First simplify the equations. Start with the first equation. Divide both sides by 2.

$4x + 2y = 24$
$2x + y = 12$

Now simplify the second equation. Multiply both sides by $2x$.

$\frac{7y}{2x} = 7$
$7y = 7(2x)$
$7y = 14x$

Divide both sides by 7.

$y = 2x$

You need to find the value of x. Substitute $2x$ for y in the first equation.

$2x + y = 12$
$2x + (2x) = 12$
$4x = 12$
$x = 3$

The answer is 3.